INTEREST, DISCOUNT,

EQUATION OF PAYMENTS,

&c.;

WITH NUMEROUS

PROBLEMS FOR SOLUTION.

BY

DANA P. COLBURN,

PRINCIPAL OF THE RHODE ISLAND STATE NORMAL SCHOOL, PROVIDENCE,
AUTHOR OF DECIMAL SYSTEM OF NUMBERS, AND CO-AUTHOR
OF FIRST STEPS IN NUMBERS.

BOSTON:
BENJAMIN B. MUSSEY & CO.,
29 CORNHILL.
1854.

CHAPTER V. OF THE LIBRARY.

The Library Committee shall divide the books and other articles belonging to the Library into three classes, namely: (a) those which are not to be removed from the building; (b) those which may be taken from the halls only by written permission of three members of the committee, who shall take a receipt for the same and be responsible for their safe return; (c) those which may circulate under the following rules:—

Members shall be entitled to take from the Library one folio, or two quarto volumes, or four volumes of any lesser fold, with the plates belonging to the same, upon having them recorded by the Librarian, or Assistant Librarian, and promising to make good any damage they sustain, while in their possession, and to replace the same if lost, or pay the sum fixed by the Library Committee.

No person shall lend any book belonging to the Institute, excepting to a member, under a penalty of one dollar for every such offence.

The Library Committee may allow members to take more than the allotted number of books upon a written application, and may also permit other persons than members to use the Library, under such conditions as they may impose.

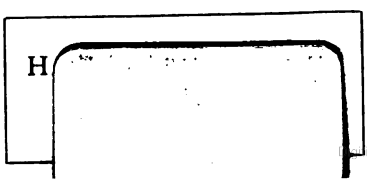
No person shall detain any book longer than four weeks from the time of its being taken from the Library, if notified that the same is wanted by another member, under a penalty of five cents per day, and no volume shall be detained longer than three months at one time under the same penalty.

The Librarian shall have power by order of the Library Committee to call in any volume after it has been retained by a member for ten days.

On or before the first Wednesday in May, all books shall be returned to the Library, and a penalty of five cents per day shall be imposed for each volume detained.

Labels designating the class to which each book belongs shall be placed upon its cover.

No book shall be allowed to circulate until one month after its reception.



INTEREST, DISCOUNT,

EQUATION OF PAYMENTS,

&c.;

WITH NUMEROUS

PROBLEMS FOR SOLUTION.

BY

DANA P. COLBURN,

PRINCIPAL OF THE RHODE ISLAND STATE NORMAL SCHOOL, PROVIDENCE,
AUTHOR OF DECIMAL SYSTEM OF NUMBERS, AND CO-AUTHOR
OF FIRST STEPS IN NUMBERS.

BOSTON:
BENJAMIN B. MUSSEY & CO.,
29 CORNHILL.
1854.

STEREOTYPED AT THE
BOSTON STEREOTYPE FOUNDRY.

PREFACE.

THIS treatise illustrates the principles involved in Interest, Equation of Payments, &c., and exhibits such methods of applying them as seem most brief and expeditious, and best adapted to the wants and circumstances of business men.

A large number of problems are inserted in order to furnish the student in school with material for practice. They are of such a character as to exhibit the nature of the most common transactions involving interest and percentage, and to require such computations as the accountant is most frequently called on to perform. Many of them cannot be solved without considerable labor; but they are not more difficult than those of a similar nature which come up in real life, and which any one fitting for business pursuits should be able to master.

As a general thing, answers to the problems are not inserted They are omitted for the following and other reasons.

1. They are unnecessary, since every example admits of rigid proof.

2. They are never given in the problems of real life.

3. A learner should become practically acquainted with those tests which alone he can apply when acting for himself; for then it will be as important for him to be sure of the truth of his results as it will be to obtain them.

4. The proof will often make an operation appear plain and simple which would otherwise have seemed obscure and complicated.

5. The proof often furnishes as valuable an arithmetical exercise as did the original solution.

6. The necessity of verifying his work for himself will lead the pupil to be more careful and accurate in performing it.

In short, it has been the aim of the author to make this treatise in every respect a practical one, and he now submits it to the public with the hope that it may be found worthy of some degree of patronage.

INTEREST, EQUATION OF PAYMENTS, &c.

§ 1. INTRODUCTORY.

When a person hires an article of property of another, it is evident that, at the expiration of the time for which he hires it, he ought to return it, and pay for its use. Moreover the sum paid for the use of the borrowed article should be proportioned both to its value, and the length of time it is kept.

For instance, if I hire two houses, one of which is worth twice as much as the other, I ought to pay twice as much per year for the first as for the second. If the values of the houses are alike, and one is kept one half as long as the other, only one half as much ought to be paid for the first as for the second.

To the Teacher. — It will be well to illustrate the above important principles by questions similar in character to the following: —

If one man hires a horse to go a certain distance, and another hires one to go twice as far, how many times as much ought the second to pay for its use as the first pays? What would have been the answer to the above question, provided the second man had gone 3 times as far as the first? 4 times as far? $3\frac{1}{2}$ times as far? $\frac{1}{2}$ as far? $\frac{5}{6}$ as far? $\frac{2}{3}$ as far? &c., &c. If the horses are hired by the hour, and the first man keeps his horse three times as many hours as the second keeps his, how many times as much ought he to pay for the use of it? What would have been the answer had he kept it 5 times as long as the second? 8 times as long? 6 times as long? $\frac{1}{2}$ as long? $\frac{3}{7}$ as long? $\frac{1}{6}$ as long? &c., &c.

Similar questions should be asked with reference to other objects hired, as houses, money, &c., &c., till the principle is fully understood.

§ 2. DEFINITIONS.

Money is very frequently hired, and the sum to be paid for its use is determined in accordance with the above principles.

Money thus paid for the use of money is called *Interest.* The money used is called the *Principal.* The principal and interest added together form the *Amount,* or entire sum due at any given time. The interest of any principal is usually reckoned as a certain *per cent,* i. e., a certain number of one hundredths of that principal, for each year it is on interest. This per cent is called the *Rate per cent,* or simply the *Rate.*

NOTE. — The term *per cent,* from the Latin *per centum,* originally meant *by the hundred,* but as it is now used in arithmetic, it means *one hundredth.* Thus 6 per cent means $\frac{6}{100}$, or .06; 4 per cent means $\frac{4}{100}$, or .04; $\frac{1}{2}$ per cent means $\frac{\frac{1}{2}}{100}$, or $\frac{1}{2}$ of $\frac{1}{100}$, or $\frac{1}{200}$, &c., &c. This term may be applied to any thing else as well as money; and hence the definition (often given in the school room) "so many cents on 100 cents" is not a good one, any more than would be, *so many bushels on* 100 *bushels, or so many yards on* 100 *yards.* It is the more objectionable because scholars are sometimes led by it, and by being called upon to use the term *per cent* only in connection with money, to suppose that it has some necessary connection with cents, or with United States money.

§ 3. LEGAL RATE.

In nearly every state of the Union, laws have been passed forbidding persons to receive interest at more than some given rate. The highest annual rate of interest which the law of any state allows is called the *Legal Rate* of that state. Any excess of interest over the legal rate is called *Usury.* Those who receive usury are liable to be fined, or punished in some other way.*

In New York, South Carolina, Michigan, Wisconsin, and Iowa, the legal rate of interest is 7 per cent per year.

In Georgia, Alabama, Mississippi, Florida, and Texas, it is 8 per cent per year.

* It may not be amiss to remark that the laws regulating the rate of interest are very often disregarded, while the penalties for their violation are seldom imposed. Very few men continue long in business without paying or receiving interest at more than the legal rate. Money, having, like every thing else, a variable value, will bring what it is worth at the time it is sold or let, and it seems as impossible to regulate by law the price which shall be paid for its use, as to fix by law that which shall be paid for the use of any other article of property.

In Louisiana, it is 5 per cent; but the banks are allowed to charge 6 per cent.

In all the other states, the legal rate is 6 per cent per year.

By special agreement of the parties, interest may be charged at the rate of 10 per cent per annum in Mississippi, Louisiana, Missouri, and Arkansas, and at the rate of 12 per cent in Illinois, Wisconsin, and Iowa. In Maryland, the interest of debts due on tobacco contracts may be reckoned at 8 per cent.

On debts in favor of the United States, interest is computed at the rate of 6 per cent.

In each state, interest is reckoned at the legal rate of that state, unless otherwise specified.

In the United States, it is customary, when reckoning interest, to regard a year as 12 months, and a month as 30 days. But in England, the year is regarded as 365 days, and the number of days in the months considered, are reckoned as in the calendar.

§ 4. INTEREST FOR 2 MONTHS, 200 MONTHS, &c., AT 6 PER CENT.

1. If the interest of any sum for one year is 6 per cent of that sum, for $\frac{1}{6}$ of 1 year, or 2 months, it must be $\frac{1}{6}$ of 6 per cent, or 1 per cent of it. Therefore, at 6 per cent per year, the interest for 2 months is 1 per cent, or .01 of the principal, and may be found by removing the decimal point of the written principal two places towards the left. Thus the interest of \$75 for 2 months is \$.75; of \$364.30 is \$3.643, &c., &c.

2. What is the interest of \$84 for 2 months? of \$687? of \$1486.70? of \$327.41?* of \$637.86? of \$429.87? of \$838.75? of \$3.86? of \$57?

3. What is the amount of each of the above sums for 2 months?

4. If the interest of any sum for 2 months is .01 of that sum for .1 of 2 months, which is 6 days, it must be .1 of .01, or .001 of it. Therefore, at 6 per cent per year, the interest for 6

* The denominations below mills need not be given in the answer. Indeed, those below cents need not be given, if, when there are more than 5 mills, 1 be added to the number of cents.

days is .001 of the principal, and may be found by removing the decimal point three places to the left. Thus the interest of $987 for 6 days is $.987; of $439 is $.439; of $8763.72 is $8.764;* or, carrying the values no farther than cents, the interest of $987 is $.99; of $439 is $.44; of 8763.72 is $8.76, &c., &c.

5. What is the interest of $586.37 for 6 days? of $396? of $28? of $67? of $36.75? of $1473.87? of $142?

6. What is the amount of each of the above sums for 6 days?

7. If the interest of any sum for 2 months is 1 per cent of that sum for 100 times 2 months, or 200 months, it must be 100 times 1 per cent, or 100 per cent of it, which is the sum itself. Therefore, at 6 per cent per year, the interest for 200 months, or 16 years 8 months, must equal the principal. Thus the interest of $47 for 200 months is $47; of $37.84 is $37.84; of $23 is $23, &c., &c.

8. What is the interest of $38.73 for 16 yr. 8 mo., or 200 mo.? of $.57? of $67.95? of $27.63? of $.28? of $1.07?

9. What is the amount of each of the above sums for 200 months?

10. If the interest of any sum for 200 months is equal to that sum for $\frac{1}{10}$ of 200 months, or 20 months, it must be $\frac{1}{10}$ of it. Therefore, at 6 per cent per year, the interest for 20 months, or 1 year and 8 months, is $\frac{1}{10}$ of the principal, and may be found by removing the decimal point one place to the left. Thus the interest of $63 for 20 months, or 1 yr. 8 mo., is $6.30; of $78.60 is $7.86; of $8.79 is $.879, or, carrying the result only to cents, $.88.

11. What is the interest of $73.86 for 20 months, or 1 yr. 8 mo.? of $578? of $673.70? of $48.63? of $5.87? of $63? of $986.20? of $37.82?

12. What is the amount of each of the above sums for 20 months?

§ 5. RECAPITULATION AND INFERENCES.

The importance of the above deductions is such as to demand that they should be made perfectly familiar by all who would

* Since $.00072 = more than ½ of a mill.

become expert in casting interest at 6 per cent. We therefore repeat them.

When interest is 6 per cent per year, —

1st. *The interest of any sum for* 200 *months, or* 16 *yr.* 8 *mo., will equal that sum.*

2d. *The interest of any sum for* 20 *months, or* 1 *yr.* 8 *mo., will equal* $\frac{1}{10}$ *of that sum, or as many dimes as there are dollars in the principal.*

3d. *The interest of any sum for* 2 *months will equal* .01 *of that sum, or as many cents as there are dollars in the principal.*

4th. *The interest of any sum for* 6 *days will equal* .001 *of that sum, or as many mills as there are dollars in the principal.*

It is therefore evident that for $\frac{1}{2}$ of 200 months, the interest will equal $\frac{1}{2}$ of the principal; for $\frac{1}{3}$ of 200 months, $\frac{1}{3}$ of the principal, &c., &c.

For $\frac{1}{2}$ of 20 months, the interest will equal $\frac{1}{2}$ of $\frac{1}{10}$, or $\frac{1}{20}$ of the principal; for $\frac{1}{3}$ of 20 mo., $\frac{1}{3}$ of $\frac{1}{10}$, or $\frac{1}{30}$ of the principal; for 3 times 20 mo., 3 times .1, or .3 of the principal, &c., &c.

For $\frac{1}{2}$ of 2 months, the interest will equal $\frac{1}{2}$ of .01, or $\frac{1}{200}$ of the principal; for $\frac{1}{4}$ of 2 months, $\frac{1}{4}$ of .01, or $\frac{1}{400}$ of the principal; for 7 times 2 months, 7 times .01, or .07 of the principal, &c., &c.

For $\frac{1}{2}$ of 6 days, the interest will be $\frac{1}{2}$ of .001, or $\frac{1}{2000}$ of the principal; for 3 times 6 days, 3 times .001, or .003 of the principal, &c., &c.

§ 6. TABLE OF ALIQUOT PARTS OF 200 MONTHS, 20 MONTHS, &c.

The following table suggests many applications of the foregoing principles: —

$\frac{1}{2}$ of 200 mo. = 100 mo. = 8 yr. 4 mo.
$\frac{1}{3}$ of 200 mo. = $66\frac{2}{3}$ mo. = 5 yr. 6 mo. 20 da.
$\frac{1}{4}$ of 200 mo. = 50 mo. = 4 yr. 2 mo.
$\frac{1}{5}$ of 200 mo. = 40 mo. = 3 yr. 4 mo.
$\frac{1}{6}$ of 200 mo. = $33\frac{1}{3}$ mo. = 2 yr. 9 mo. 10 da.

$\frac{1}{6}$ of 200 mo. = 25 mo. = 2 yr. 1 mo.
$\frac{1}{10}$ of 200 mo. = 20 mo. = 1 yr. 8 mo.
$\frac{1}{12}$ of 200 mo. = $16\frac{2}{3}$ mo. = 1 yr. 4 mo. 20 da.
$\frac{1}{15}$ of 200 mo. = $13\frac{1}{3}$ mo. = 1 yr. 1 mo. 10 da.
$\frac{1}{16}$ of 200 mo. = $12\frac{1}{2}$ mo. = 1 yr. 15 da.
$\frac{1}{2}$ of 20 mo. = 10 mo.
$\frac{1}{3}$ of 20 mo. = $6\frac{2}{3}$ mo. = 6 mo. 20 da.
$\frac{1}{4}$ of 20 mo. = 5 mo.
$\frac{1}{5}$ of 20 mo. = 4 mo.
$\frac{1}{6}$ of 20 mo. = $3\frac{1}{3}$ mo. = 3 mo. 10 da.
$\frac{1}{8}$ of 20 mo. = $2\frac{1}{2}$ mo. = 2 mo. 15 da.
$\frac{1}{10}$ of 20 mo. = 2 mo.
$\frac{1}{12}$ of 20 mo. = $1\frac{2}{3}$ mo. = 1 mo. 20 da.
$\frac{1}{15}$ of 20 mo. = $1\frac{1}{3}$ mo. = 1 mo. 10 da.
$\frac{1}{2}$ of 2 mo. = 1 mo.
$\frac{1}{3}$ of 2 mo. = $\frac{2}{3}$ of 1 mo. = 20 da.
$\frac{1}{4}$ of 2 mo. = $\frac{1}{2}$ of 1 mo. = 15 da.
$\frac{1}{5}$ of 2 mo. = $\frac{2}{5}$ of 1 mo. = 12 da.
$\frac{1}{6}$ of 2 mo. = $\frac{1}{3}$ of 1 mo. = 10 da.
$\frac{1}{10}$ of 2 mo. = $\frac{1}{5}$ of 1 mo. = 6 da.
$\frac{1}{12}$ of 2 mo. = $\frac{1}{6}$ of 1 mo. = 5 da.
$\frac{1}{15}$ of 2 mo. = $\frac{2}{15}$ of 1 mo. = 4 da.
$\frac{1}{2}$ of 6 da. = 3 da., $\frac{1}{3}$ of 6 da. = &c., &c.

§ 7. QUESTIONS TO INSURE READINESS IN APPLICATION OF PRINCIPLES.

1. At 6 per cent per year, what part of the principal will be equal to the interest for 50 mo.?
2. For 100 mo.? for 40 mo.? for 4 yr. 2 mo.?
3. For 25 mo.? for $33\frac{1}{3}$ mo.? for 3 yr. 4 mo.?
4. For 1 yr. 15 da.? for $16\frac{2}{3}$ mo.? for 8 yr. 4 mo.?
5. For 16 yr. 8 mo.? for 2 yr. 1 mo.? for 1 yr. 8 mo.?
6. For 1 yr. 4 mo. 20 da.? for $66\frac{2}{3}$ mo.? for $13\frac{1}{3}$ mo.?
7. For 2 yr. 9 mo. 10 da.? for 5 yr. 6 mo. 20 da.? for $12\frac{1}{2}$ mo.?

8. At 6 per cent per year, what part of the principal will be equal to the interest for 6 mo. 20 days?

Solution. — Since 6 mo. 20 da. $=\frac{1}{3}$ of 20 mo., the required interest must be $\frac{1}{3}$ of $\frac{1}{10}$, or $\frac{1}{30}$ of the principal.

9. At 6 per cent per year, what part of the principal will be equal to the interest for 10 mo.? for 4 mo.? for 5 mo.?

10. For 2 mo.? for $1\frac{1}{2}$ mo.? for $3\frac{1}{3}$ mo.?

11. For $2\frac{1}{2}$ mo.? for $1\frac{2}{3}$ mo.? for 1 mo. 10 da.?

12. For 1 mo. 20 da.? for 2 mo. 15 da.? for 3 mo. 10 da.?

13. At 6 per cent per year, what part of the principal will be equal to the interest for 15 da.?

Solution. — Since 15 da. $=\frac{1}{2}$ of 2 mo., the required interest must be $\frac{1}{2}$ of $\frac{1}{100}$, or $\frac{1}{200}$ of the principal.

14. At 6 per cent per year, what part of the principal will be equal to the interest for 20 da.? for 1 mo., or 30 da.?

15. For 6 da.? for 12 da.? for 15 da.? for 4 da.?

16. For 5 da.? for 10 da.? for 3 da.? for 1 da.?

17. How long must a principal be on interest that the interest may equal $\frac{1}{2}$ of it? $\frac{1}{3}$ of it?

18. $\frac{1}{10}$ of it? $\frac{1}{15}$ of it? $\frac{1}{8}$ of it? $\frac{1}{4}$ of it?

19. $\frac{1}{16}$ of it? $\frac{1}{12}$ of it? $\frac{1}{6}$ of it? $\frac{1}{5}$ of it?

20. $\frac{1}{2}$ of $\frac{1}{10}$ of it? $\frac{1}{3}$ of $\frac{1}{10}$ of it? $\frac{1}{6}$ of $\frac{1}{10}$ of it?

21. $\frac{1}{5}$ of $\frac{1}{10}$ of it? $\frac{1}{4}$ of $\frac{1}{10}$ of it? $\frac{1}{8}$ of $\frac{1}{10}$ of it?

22. $\frac{1}{20}$ of it? $\frac{1}{30}$ of it? $\frac{1}{60}$ of it? $\frac{1}{50}$ of it?

23. $\frac{1}{40}$ of it? $\frac{1}{12}$ of $\frac{1}{10}$ of it? $\frac{1}{120}$ of it?

24. $\frac{1}{15}$ of $\frac{1}{10}$ of it? $\frac{1}{150}$ of it? $\frac{1}{60}$ of it?

25. .01 of it? $\frac{1}{2}$ of .01 of it? $\frac{1}{200}$ of it?

26. $\frac{1}{3}$ of .01 of it? $\frac{1}{500}$ of it? $\frac{1}{6}$ of .01 of it?

27. $\frac{1}{600}$ of it? $\frac{1}{300}$ of it? $\frac{1}{1200}$ of it? $\frac{1}{1500}$ of it?

28. .001 of it? $\frac{1}{2}$ of $\frac{1}{1000}$ of it? $\frac{1}{2000}$ of it?

29. $\frac{1}{3000}$ of it? $\frac{1}{6}$ of .001 of it? $\frac{1}{6000}$ of it?

§ 8. INTEREST FOR CONVENIENT TIMES.

1. What is the interest of $156.96 for 1 yr. 4 mo. 20 da.?

Solution. — Since at 6 per cent interest for 200 mo. equals

the principal, and since 1 yr. 4 mo. 20 da. equals $\frac{1}{12}$ of 200 mo., the required interest must equal $\frac{1}{12}$ of the given principal; that is, $\frac{1}{12}$ of \$156.96, which is \$13.08 = Ans.

2. What is the interest of \$36 for 3 yr. 4 mo.? for 2 yr. 9 mo. 10 da.?

3. What is the interest of \$24.72 for $16\frac{2}{3}$ mo.? for 5 yr. 6 mo. 20 da.?

4. What is the interest of \$144.24 for 1 yr. 1 mo. 10 da.? for $33\frac{1}{3}$ mo.?

5. What is the interest of \$231.12 for 1 yr. 15 da.? for 4 yr. 2 mo.?

6. What is the interest of \$1728 for 1 yr. 4 mo. 20 da.? for 2 yr. 1 mo.?

7. What is the interest of \$500 for $66\frac{2}{3}$ mo.? for 16 yr. 8 mo.?

8. What is the interest of \$42.24 for 1 yr. 8 mo.? for 10 mo.?

9. What is the interest of \$150 for 6 mo. 20 da.? for 3 mo. 10 da.?

10. What is the interest of \$4.80 for 5 mo.? for 4 mo.? for $1\frac{2}{3}$ mo.?

11. What is the interest of \$17.70 for 2 mo. 15 da.? for 1 mo. 10 da.?

12. What is the interest of \$87.18 for 1 mo. 20 da.? for 6 mo. 20 da.?

13. What is the interest of \$537 for 2 mo.? for 1 mo.? for 15 da.?

14. What is the interest of \$288 for 10 da.? for 5 da.? for 12 da.?

15. What is the interest of \$96.34 for 20 da.? for 6 da.? for 5 da.?

16. What is the interest of \$675 for 2 da.? for 3 da.? for 1 da.?

17. What is the interest of \$438.74 for 3 yr. 4 mo.? for 2 yr. 9 mo. 10 da.?

18. What is the interest of \$73.87 for 1 yr. 15 da.? for 6 mo. 20 da.?

§ 9. INTEREST FOR VARIOUS TIMES.

In computing interest for other times than those already mentioned, it is usually most convenient to divide the time into parts, as illustrated below. The letters a, b, c, &c., are inserted merely for convenience of reference, in indicating how the numbers opposite to which they stand have been obtained. Thus, in first solution, $\frac{1}{30}$ of a = b = \$6.557 =, &c., indicates that \$6.557 is $\frac{1}{30}$ of the principal. b + c = \$8.524 =, &c., indicates that \$8.524 is the sum of \$6.557 and \$1.967, the numbers standing opposite b and c.

1. What is the interest of \$196.72 for 8 mo. 20 da.?

1st Solution.

a = \$196.72 = Principal.

$\frac{1}{30}$ of a = b = 6.557 = Int. for 6 mo. 20 da.
.01 of a = c = 1.967 = Int. for 2 mo.
b + c = \$ 8.524 = Int. for 8 mo. 20 da.

2d Solution.

a = \$196.72 = Principal.

.04 of a = b = 7.869 = Int. for 8 mo.
$\frac{1}{300}$ of a = c = .655 = Int. for 20 da.
b + c = \$ 8.524 = Int. for 8 mo. 20.

3d Solution.

a = \$196.72 = Principal.

$\frac{1}{20}$ of a = b = 9.836 = Int. for 10 mo.
$\frac{1}{150}$ of a = c = 1.311 = Int. for 1 mo. 10 da.
b − c = \$ 8.525 = Int. for 8 mo. 20 da.

2. What is the amount of \$649.37 for 17 mo. 15 da.?

1st Solution.

a = \$649.37 = Principal.

$\frac{1}{12}$ of a = b = \$ 54.114 = Int. for 16 mo. 20 da.
* $\frac{1}{20}$ of b = c = \$ 2.705 = Int. for 25 da.
a + b + c = \$706.189 = Amt. 17 mo. 15 da.

* Since 16 mo. 20 da. = 500 da.

2d Solution.

$a = \$649.37$ = Principal.
$\frac{1}{10}$ of $a = b =$ 40.585 = Int. for 12 mo. 15 da.
$\frac{1}{40}$ of $a = c =$ 16.234 = Int. for 5 mo.
$a + b + c = \$706.189$ = Amt. for 17 mo. 15 da.

3d Solution.

$a = \$649.37$ = Principal.
$\frac{1}{10}$ of $a = b =$ 64.937 = Int. for 20 mo.
$\frac{1}{8}$ of $b = c =$ 8.117 = Int. for 2 mo. 15 da.
$a + b - c = \$706.190$ = Amt. for 17 mo. 15 da.

4th Solution.

$a = \$649.37$ = Principal.
$\frac{1}{20}$ of $a = b =$ 32.468 = Int. for 10 mo.
$\frac{1}{4}$ of a, or $\frac{1}{2}$ of $b, = c =$ 16.234 = Int. for 5 mo.
$\frac{1}{8}$ of a, or $\frac{1}{2}$ of $c, = d =$ 8.117 = Int. for 2 mo. 15 da.
$a + b + c + d = \$706.189$ = Amt. for 17 mo. 15 da.

NOTE. — Many other solutions might have been given to the above questions, but as they would all involve similar principles, it is unnecessary to add them. Every question in interest admits a great variety of solutions, and the pupil should examine it carefully to determine which he will adopt. Practice will enable him to select a good method at once. One process may be applied to test the correctness of a result obtained by some other. We may remark, that, as a general thing, it is better to divide than to multiply, for in division we have to consider no denomination below the lowest we wish to have in the answer.

3. What is the interest of \$857.63 for 3 mo. 16 da.?
4. What is the interest of \$875.37 for 1 mo. 26 da.?
5. What is the interest of \$93.75 for 9 mo. 29 da.?
6. What is the interest of \$178.43 for 16 mo. 14 da.?
7. What is the interest of \$343.65 for 13 mo. 16 da.?
8. What is the interest of \$237.64 for 19 mo. 24 da.?
9. What is the interest of \$478.96 for 17 mo. 26 da.?
10. What is the interest of \$375.81 for 22 mo. 15 da.?
11. What is the interest of \$58.27 for 96 mo. 20 da.?

12. What is the interest of $5789.00 for 29 mo. 29 da.?
13. What is the interest of $80.32 for 7 mo. 19 da.?
14. What is the interest of $175.00 for 1 mo. 17 da.?
15. What is the interest of $326.00 for 8 mo. 23 da.?
16. What is the interest of $27.96 for 1 yr. 3 mo. 13 da.?
17. What is the interest of $578.31 for 3 yr. 7 mo. 28 da.?
18. What is the interest of $724.16 for 7 yr. 2 mo. 11 da.?
19. What is the interest of $4369.87 for 3 mo. 26 da.?
20. What is the amount of $25.50 for 9 mo. 27 da.?
21. What is the amount of $117.58 for 3 yr. 1 mo. 18 da.?
22. What is the amount of $313.27 for 6 mo. 9 da.?
23. What is the amount of $57.75 for 9 mo. 1 da.?
24. What is the amount of $35.86 for 11 mo. 25 da.?
25. What is the amount of $17.64 for 1 yr. 1 mo. 13 da.?
26. What is the amount of $378.51 for 1 yr. 5 mo. 17 da.?
27. What is the amount of $632.87 for 2 yr. 11 mo. 13 da.?

§ 10. COMPUTATION OF TIME, AND APPLICATION TO PROBLEMS.

In business transactions, it is usually necessary to compute the time during which money has been on interest; that is, the time between the dates on which interest began and ended. The usual method of doing this is to reckon the number of entire years, then the number of entire calendar months remaining, and then the remaining days. The year is (as before) regarded as 360 days, or 12 months of 30 days each, and the entire calendar months as months of 30 days, but the days which are left after reckoning the years and months are determined by counting them according to the number in the months in which they occur.

1. What is the time from Jan. 17, 1845, to June 28, 1849?

Solution. — From Jan. 17, 1845, to Jan. 17, 1849, is 4 years; from Jan. 17 to June 17 is 5 months; from June 17 to June 28 is 11 days. Therefore the required time is 4 yr. 5 mo. 11 da.

2. What is the time from April 27, 1846, to Feb. 13, 1851?

Solution. — From April 27, 1846, to April 27, 1850, is 4 years; from April 27, 1850, to Jan. 27, 1851, is 9 months; January having 31 days, there are 4 days left in it, which, added to the 13 in February, give 17 days. Therefore the required time is 4 yr. 9 mo. 17 da.

3. What is the time from Sept. 24, 1849, to March 20, 1852?

Solution. — From Sept. 24, 1849, to Sept. 24, 1851, is 2 years; from Sept. 24, 1851, to Feb. 24, 1852, is 5 months; 1852 being leap year, February has 29 days; hence there are 5 days left in it, which, added to the 20 in March, give 25 days. Therefore the required time is 2 yr. 5 mo. 25 da.

NOTE. — This method of computing the time, though the one usually adopted by business men when interest is computed for months and days, is unequal in its operation; for the calendar months, though varying in length from 28 to 31 days, are all reckoned as months of 30 days each. Hence the interest of a sum during the month of February will be as much as during either March or April, though February contains 3 days less than March, and 2 less than April.

By this method, the interest on four notes dated respectively on the 28th, 29th, 30th, and 31st of any one month, and paid on any one day between the 1st and 28th of March, of any year, except leap year, would be computed for the same time. Suppose, for instance, that they are dated in October, 1850, and paid March 15, 1851. Then for the first note dated Oct. 28, the time will be found without difficulty to be 4 mo. 15 da. In calculating the time on the others, we proceed thus: Since there are not as many as 29 days in February, 1851, we reckon from the 29th, 30th, or 31st of Oct. to the last day of February as 4 months, to which adding the 15 days in March gives 4 months and 15 days as the time, in each case. The restriction with reference to notes paid in leap year is necessary simply because February has then 29 days. The proposition will *always* be true of notes dated on the 29th, 30th, and 31st, of any month, and paid at any time between the 1st and 28th of March.

Again: four notes dated respectively on the 28th, 29th, 30th, and 31st of August of any year except the one immediately preceding leap year, and payable in 6 months, would all become due on the same day.

The only strictly accurate method of reckoning time is to actually count the days in each month we consider. Thus to reckon the time from Oct. 28, 1850, to March 15, 1851, we proceed as follows: From Oct. 28th to 31st, is 3 days; to which adding the 30 days in November, the 31 in Decem-

ber, the 31 in January, the 28 in February, and the 15 in March, gives 138 days as the true time between the two dates. In England the time is always computed in this way, as it is in this country when notes are payable at the end of a certain number of *days*.

4. What is the time from June 23, 1850, to June 3, 1852? Ans. 1 yr. 11 mo. 11 da.

5. What is the time from May 18, 1847, to Oct. 8, 1851?

6. What is the time from Jan. 31, 1851, to March 23, 1852?

7. What is the time from Nov. 17, 1849, to Dec. 12, 1851?

8. What is the time from Dec. 31, 1848, to July 6, 1850?

9. What is the interest of $787.36 from May 3, 1843, to Dec. 17, 1845? $123.87

10. What is the interest of $54.76 from Feb. 14, 1840, to June 2, 1844? $14.15 5 1 mo 19 days

11. What is the amount of $476.35 from June 30, 1847, to Dec. 28, 1850? $576.22 4 1 mo 28 days

12. What is the amount of $638.29 from May 31, 1851, to Oct. 7, 1852? $ 670.89 16 mo 7 days

13. What is the amount of $4937.56 from Dec. 19, 1843, to Feb. 16, 1847?

14. What is the interest of $481.74 from Jan. 29, 1847, to March 25, 1851? $120.03 4 yr 2 mo 25

15. What is the interest of $587.60 from Jan. 31, 1850, to July 18, 1852? $86.76 2 yr 5 mo 18 days

16. What is the amount of $947.84 from May 15, 1850, to June 13, 1851? $1004.24 1

17. What is the interest of $748.67 from Dec. 14, 1849, to May 4, 1851? $62.38

18. What is the interest of $1546.61 from April 9, 1847, to June 1, 1851? $334.85

19. What is the interest of $917.68 from June 5, 1842, to Jan. 1, 1850? $417.6 5

20. What is the amount of $8396.58 from April 30, 1847, to March 22, 1850? $698.54 7 16

21. What is the amount of $1449.13 from Dec. 31, 1850, to March 5, 1852? $1551.176

22. What is the amount of $787.31 from Sept. 8, 1849, to July 1, 1851? $873.65

23. What is the interest of $6854.13 from July 17, 1852, to Sept. 29, 1852? $82.247

24. Jan. 15, 1852, George W. Pratt borrowed $237.50 of A. N. Johnson, and Feb. 13, 1852, he borrowed $438.75 more, agreeing to pay interest at 6 per cent per year. He paid the debts March 8, 1852. What was their amount? $680.05

25. I have three notes against Arthur Sumner, viz., one for $548.17, dated Jan. 1, 1851, another for $679.18, dated Jan. 27, 1852, and another for $376.89, dated May 31, 1852. What amount will be due on all of them July 8, 1852? $1614.82

26. Jan. 1, 1851, I borrowed $3468, with which I purchased flour at $6 per barrel. I sold the flour March 17, 1851, for $6.50 per barrel, cash. Interest being reckoned at 6 per cent, did I gain or lose by the transaction, and how much? $21.51

27. Sept. 3, 1847, I borrowed $594, with which I purchased cloth at $3 per yard; Oct. 11, 1847, I borrowed $1352, with which I purchased cloth at $4 per yard. Nov. 1, 1847, I sold both lots at $4 per yard, cash, and immediately paid the amount of the borrowed money at 6 per cent. Did I gain or lose, and how much?

28. April 16, 1850, I bought of Mr. Curry 498 cords of wood at $3.50 per cord, giving in payment my note payable on demand with interest. Oct. 5, 1850, I sold 232 cords of it at $3.75 per cord, cash, and immediately lent the money received for it on interest. Oct. 17, 1850, I sold the remainder at $4.07 per cord, to be paid Jan. 1, 1851, and to be on interest after Nov. 1, 1850. Jan. 1, 1851, I collected the money due me, and paid that due to Mr. Curry. How much did I gain or lose by the transactions?

§ 11. INTEREST BY DAYS.

Many business men always reduce the time to days, and compute the interest by the method illustrated below. As a general thing, however, this method is not so convenient as the preceding.

Since, at 6 per cent per year, the interest for 1 day is $\frac{1}{6}$ of $\frac{1}{1000}$ of the principal, it follows that the interest for any number of days must be $\frac{1}{6}$ as many thousandths of the principal as there are days. We may, therefore, find the interest for any number of days, by multiplying the principal by $\frac{1}{6}$ of the number of days, and removing the point three places farther to the left. It will make no difference with the result, whether we multiply by $\frac{1}{6}$ of the number of days, or by the number of days and divide by 6, but it will usually be better to divide before multiplying.

1. What is the interest of $437.62 for 4 mo. 3 da.?

Solution. — Since 4 mo. 3 da. = 123 da., the required interest must be $\frac{1}{6}$ of .123 of the principal, which is .020$\frac{1}{2}$ of the principal. The work carried to mills would be written thus: —

$437.62
.020½
8.752
.218
$8.970 = Ans.

2. What is the interest of $281.87 for 5 mo. 9 da. at 6 per cent?
3. What is the interest of $581.21 for 6 mo. 24 da.? $19.76
4. What is the interest of $1764.25 for 1 yr. 1 mo. 16 da.? $119.38
5. What is the interest of $83.25 for 2 mo. 21 da.? $1.12
6. What is the interest of $98.37 for 6 mo. 18 da.? $3.25
7. What is the interest of $54.57 for 4 mo. 20 da.? $1.27
8. What is the interest of $397.42 for 8 mo. 15 da.? $16.87
9. What is the interest of $231.48 for 7 mo. 12 da.? $9.56
10. What is the interest of $438.64 for 5 mo. 24 da.? $12.72

§ 12. INTEREST BY DOLLARS, FOR MONTHS AND CONVENIENT PARTS OF A MONTH.

We can frequently compute interest with great ease by first finding the interest for 1 day, 1 month, or 1 year, and getting the required interest from this. When the interest for any of the above times is any convenient sum, as 1 dollar, 1 dime, 1

cent, 1 mill, $\frac{1}{2}$ dollar, $\frac{1}{3}$ dollar, &c., the proposed course will be particularly advantageous.

The previous examples have shown that, at 6 per cent per year, the interest of any sum for 1 month is $\frac{1}{200}$ of that sum. Therefore we have the following: —

At 6 per cent per year, —

1. *The interest of* $200 *is* $1 *per month.*
2. *The interest of* $20 *is* $.10, *or* 1 *dime, per month.*
3. *The interest of* $2 *is* $.01 *per month.*

1. What is the interest of $200 for 5 months? 8 mo.? 1 yr., or 12 mo.? 2 yr. 1 mo., or 25 mo.? 2 yr. 8 mo.? 3 yr. 7 mo.?

2. What is the interest of $200 for 7 mo. 15 da.?

Solution. — The interest of $200 is $1 per month; therefore for 7 mo. 15 da., or $7\frac{1}{2}$ mo., it must be $7\frac{1}{2}$ dollars, or $7.50.

3. What is the interest of $200 for 5 mo. 10 da.? 11 mo. 6 da.? 2 yr. 8 mo. 12 da.? 3 yr. 7 mo. 25 da.?

4. What is the interest of $20 for 3 mo.? 7 mo.? 1 yr.? 1 yr. 7 mo.? 2 yr. 5 mo.? 4 mo. 15 da.? 9 mo. 5 da.? 7 mo. 20 da.?

5. What is the interest of $2 for 9 mo.? 1 yr. 7 mo.? 3 yr. 5 mo.? 4 yr. 2 mo.? 6 mo. 15 da.? 3 yr. 20 da.? 2 yr. 2 mo. 15 da.?

6. What is the interest of $100 for 7 mo.?

Solution. — Since the interest of $200 is 1 dollar per month, the interest of $100 must be a half dollar per month, and 7 half dollars, or $3.50 for 7 months.

7. What is the interest of $100 for 9 mo.? 1 yr.? 1 yr. 7 mo.? 3 yr. 4 mo.? 15 da.? 3 da.? 20 da.? 3 mo. 15 da.? 9 mo. 10 da.? 8 mo. 15 da.?

8. What is the interest of $50 for 1 mo.? 8 mo.? 12 mo.? 17 mo.? 2 yr. 11 mo.? 15 da.? 8 mo. 15 da.? 20 da.? 4 mo. 20 da.?

9. What is the interest of $25 for 1 mo.? 4 mo.? 3 yr. 4 mo.? 9 mo.? 7 mo. 15 da.? 9 mo. 10 da.?

10. What is the interest of $150 for 1 mo.? 8 mo.? 12 mo.? 9 mo.? 17 mo.? 10 mo. 15 da.?

11. What is the interest of $400 for 1 mo.? 3 mo.? 5 mo.? 7 mo. 15 da.? 18 mo. 20 da.? 15 mo. 10 da.?

12. What is the interest of $700 for 1 mo.? 4 mo.? 1 yr. 2 mo.? 3 yr. 8 mo.? 8 mo. 5 da.?

13. What is the interest of $10 for 1 mo.? 6 mo.? 9 mo.? 13 mo.? 5 yr. 4 mo.?

14. What is the interest of $1 for 1 mo.? 10 mo.? 1 yr. 6 mo.? 2 yr. 4 mo.? 3 yr. 7 mo.?

§ 13. INTEREST BY DOLLARS, WHEN THE TIME IS IN DAYS, OR MONTHS AND DAYS.

Since the interest of any sum for 6 days is .001 of that sum, the interest of 1 dollar for 6 days must be .001 of 1 dollar, which is 1 mill. If the interest of 1 dollar for 6 days is 1 mill, its interest for 1 day must be $\frac{1}{6}$ of 1 mill. Therefore the interest of $1 is $\frac{1}{6}$ of 1 mill per day.

From this we have the following statements:—

At 6 per cent.

1. *The interest of* $6 *is* 1 *mill per day.*
2. *The interest of* $60 *is* 1 *cent per day.*
3. *The interest of* $600 *is* 1 *dime per day.*
4. *The interest of* $6000 *is* 1 *dollar per day.*

NOTE. — When the interest is required for months and days, we may reduce the months to days, and then apply the above; or we may find the interest for the months, as in the last article, and then find it for the days, as above. The pupil should remember that the interest of $6 is 3 cents per month, of $60 is 3 dimes per month, and of $600 is 3 dollars per month.

1. What is the interest of $6 for 17 days? 15 da.? 27 da.? 1 mo. 3 da.? 3 mo. 7 da.? 11 mo. 13 da.?

2. What is the interest of $60 for 5 da.? 11 da.? 24 da.? 1 mo. 13 da.? 7 mo. 16 da.? 5 mo. 19 da.? 2 yr. 3 mo. 7 da.?

3. What is the interest of $600 for 7 da.? 9 da.? 1 mo. 9 da.? 3 mo. 18 da.? 6 mo. 23 da.? 1 yr. 3 mo. 16 da.? 4 yr. 7 mo. 28 da.?

4. What is the interest of $6000 for 8 da.? 19 da.? 27 da.? 2 mo. 17 da.? 15 mo. 18 da.? 2 yr. 4 mo. 11 da.? 6 yr. 3 mo. 5 da.?

5. What is the interest per day of $750?

Solution. — The interest of $6000 being 1 dollar per day, the interest of $750, which is $\frac{1}{8}$ of $6000, must be $\frac{1}{8}$ of 1 dollar per day.

6. What is the interest per day of $3000? of $2000? of $1500? of $1000? of $1200? of $12000? of $18000? *

7. What is the interest per month of each of the above sums?

8. What is the interest per day of $600? of $200? of $150? of $120? of $100? of $75? of $50? of $900? of $1200? of $1500? of $800? of $450? †

9. What is the interest per month of each of the above sums?

10. What is the interest per day of $60? of $30? of $10? of $20? of $12? of $15? of $5? of $90? of $120? of $210? of $420?

11. What is the interest per month of each of the above sums?

12. What is the interest per day of $6? of $12? of $3? of $2? of $1? of $18? of $9? of $8? of $21?

13. What is the interest per month of each of the above sums?

14. What is the interest of $3000 for 8 da.? 15 da.? 19 da.?

15. What is the interest of $1000 for 18 da.? 24 da.? 15 da.?

16. What is the interest of $12000 for 9 da.? 11 da.? 29 da.?

17. What is the interest of $30 for 16 da.? 23 da.? 27 da.?

18. What is the interest of $1500 for 12 da.? 20 da.? 28 da.?

19. What is the interest of $240 for 9 da.? 13 da.? 17 da.?

20. What is the interest of $150 for 16 da.? 19 da.? 23 da.?

21. What is the interest of $600 for 5 mo.? 17 mo.? 23 mo.?

22. What is the interest of $300 for 7 mo.? 11 mo.? 18 mo.?

23. What is the interest of $9000 for 8 mo.? 6 mo.? 12 mo.?

24. What is the interest of $30 for 7 mo. 18 da.? 4 mo. 24 da.?

25. What is the interest of $450 for 7 mo. 13 da.? 8 mo. 13 da.?

26. What is the interest of $18 for 4 mo. 23 da.? 7 mo. 2 da.?

27. What is the interest of $100 for 11 mo. 8 da.? 19 mo. 19 da.?

---

* The answers should first be given in dollars and fractions of a dollar, and then in dollars and cents.

† The answers should first be given in dimes and parts of a dime, and then in dollars and cents.

§ 14. WHEN TO DISREGARD CENTS.

If the time is not very long, the interest of any sum less than a dollar can be computed with sufficient accuracy by referring the principal to the nearest convenient aliquot part of a dollar. Thus the interest of 24 cents, for any ordinary time of calculating interest, will differ but a trifle from that of 25 cents, or $\frac{1}{4}$ of a dollar, for the same time; the interest of 35 cents will differ but a trifle from that of $33\frac{1}{3}$ cents, or $\frac{1}{3}$ of a dollar, &c. The pupil should remember that the interest of 1 dollar for 1 month, at 6 per cent per year, is $\frac{1}{2}$ of 1 cent, and hence the interest per month of 50 cents is $\frac{1}{2}$ of $\frac{1}{2}$, or $\frac{1}{4}$ of 1 cent; of 25 cents is $\frac{1}{4}$ of $\frac{1}{2}$, or $\frac{1}{8}$ of 1 cent; of $16\frac{2}{3}$ cents is $\frac{1}{6}$ of $\frac{1}{2}$, or $\frac{1}{12}$ of 1 cent; of 75 cents is $\frac{3}{4}$ of $\frac{1}{2}$, or $\frac{3}{8}$ of 1 cent, &c.

1. What is the interest of $599.77 for 9 mo. 17 da.?

Solution. $599.77 = $600 — 23 cents. The interest of $600 for 9 mo. 17 da. (being $3 per month, and 1 dime per day) is $28.70. As 23 cents is very near 25 cents, its interest must be very near $\frac{1}{4}$ of a cent per month, which will be about 1 cent for 9 mo. 17 da. This taken from $28.70 leaves $28.69 as the interest required.

2. What is the interest of $60.49 for 5 mo. 11 da.?
3. What is the interest of $59.67 for 8 mo. 13 da.?
4. What is the interest of $299.51 for 11 mo. 23 da.?
5. What is the interest of $150.32 for 7 mo. 19 da.?
6. What is the interest of $6.24 for 9 mo. 5 da.?
7. What is the interest of $200.42 for 8 mo. 15 da.?
8. What is the interest of $599.66 for 5 mo. 13 da.?
9. What is the interest of $119.94 for 13 mo. 7 da.?
10. What is the interest of $50.31 for 3 mo. 23 da.?

§ 15. INTEREST AT VARIOUS RATES, OBTAINED FROM THAT AT 6 PER CENT.

When the interest is other than 6 per cent per year, we may first find the interest at 6 per cent, and then take such part of this as the given rate is of 6 per cent. Thus the interest of any sum at 8 per cent is $\frac{8}{6} = \frac{4}{3} = 1\frac{1}{3}$ times its interest at

6 per cent; at $4\frac{1}{2}$ per cent the interest is $\frac{4\frac{1}{2}}{6} = \frac{3}{4}$ of the interest at 6 per cent, &c.

1. What is the interest of \$367.32 for 1 yr. 9 mo. 20 da. at $7\frac{1}{2}$ per cent?

Solution.

a =	\$367.32	= principal.
$\frac{1}{10}$ of a = b =	36.732	= int. 20 mo. at 6 per cent.
$\frac{1}{12}$ of b = c =	3.061	= int. 1 mo. 20 da. at 6 per cent.
b + c = d =	39.793	= int. 21 mo. 20 da. at 6 per cent.
$\frac{1}{4}$ of d = e =	9.948	= int. 21 mo. 20 da. at $1\frac{1}{2}$ per cent.
d + e =	\$49.741	= int. 1 yr. 9 mo. 20 da. at $7\frac{1}{2}$ per cent = Ans.

2. What is the interest of \$847.38 for 2 yr. 4 mo. 10 da. at 5 per cent?

3. What is the interest of \$483.94 for 3 yr. 5 mo. 26 da. at 7 per cent?

4. What is the interest of \$150 for 8 mo. 27 da. at 9 per cent?

5. What is the interest of \$46.38 for 11 mo. 19 da. at 3 per cent?

6. What is the interest of \$512.59 for 4 yr. 7 mo. 17 da. at $6\frac{1}{2}$ per cent?

7. What is the interest of \$437.95 from June 17, 1848, to May 19, 1850, at $6\frac{3}{4}$ per cent?

8. What is the interest of \$978.31 from Jan. 27, 1850, to Sept. 5, 1852, at 5 per cent?

9. What is the interest of \$87.63 from April 21, 1848, to Jan. 7, 1852, at $5\frac{1}{2}$ per cent?

NOTE. — The work may frequently be facilitated by observing that the interest of any sum at other than 6 per cent is equal to the interest at 6 per cent of the same part of that sum that the required rate is of 6 per cent. Thus the interest of any sum for a given time at 3 per cent is equal to the interest of $\frac{3}{6}$ or $\frac{1}{2}$ of that sum for the same time at 6 per cent. The interest of any sum at $4\frac{1}{2}$ per cent is equal to the interest of $\frac{4\frac{1}{2}}{6}$ or $\frac{3}{4}$ of that sum at 6 per cent, &c.

Again, the interest of any sum at other than 6 per cent is equal to its interest at 6 per cent for the same part of the given time that the required rate is of 6 per cent. Thus the interest of any sum for a given time at 2 per cent is equal to its interest for $\frac{2}{6}$ or $\frac{1}{3}$ of the given time at 6 per cent.

10. What is the interest of $450 from Aug. 12, 1852, to Oct. 7, 1852, at 7½ per cent? $5.25

11. What is the interest of $75 from Nov. 5, 1850, to Jan. 4, 1852, at 3 per cent? $2.62

12. What is the interest of $240 from Sept. 30, 1848, to May 23, 1851, at 1½ per cent? $5.03

13. What is the interest of $738.25 from Oct. 9, 1851, to July 3, 1852, at 3 per cent? $16.24

14. What is the interest of $548.39 from March 1, 1849, to Dec. 25, 1849, at 2 per cent? $5.75

§ 16. INTEREST AT VARIOUS RATES, OBTAINED DIRECTLY.

Methods similar in character to those illustrated in the following examples and explanations will usually be more brief than the preceding.

1. What is the interest of $549.84 for 1 yr. 4 mo. 15 da. at 8 per cent?

Solution.

a = $549.84 = principal.
.08 of a = b = 43.987 = int. for 1 yr. at 8 per cent.
⅓ of b = c = 14.662 = int. for 4 mo. at 8 per cent.
⅛ of c = d = 1.833 = int. for 15 da. at 8 per cent.
b + c + d = $60.482 = int. for 1 yr. 4 mo. 15 da. at 8 per cent = Ans.

2d Solution. — Since the rate is 8 per cent of the principal, the interest for 1¼ years, or 15 months, must be 1¼ times 8 per cent = 10 per cent = $\frac{1}{10}$ of the principal. Hence,

a = $549.84 = principal.
$\frac{1}{10}$ of a = b = 54.984 = int. for 15 mo. at 8 per cent.
$\frac{1}{10}$ of b = c = 5.498 = int. for 1 mo. 15 da. at 8 per cent.
b + c = $60.482 = int. for 16 mo. 15 da. at 8 per cent = Ans.

C

2. What is the interest of $537.48 for 3 yr. 8 mo. 15 da. at 7½ per cent?

Solution. — The interest for 2 yr. at 7½ per cent must be 2 times 7½, or 15 per cent of the principal. Hence we have the following written work.

	a = $537.48	principal.
.15 of a = b =	80.622	= interest 2 yr.
½ of b = c =	40.311	= interest 1 yr.
⅓ of b = d =	26.874	= interest 8 mo.
$\frac{1}{10}$ of d = e =	1.679	= interest 15 da.
b + c + d + e =	$149.486	= interest 3 yr. 8 mo. 15 da. at 7½ per cent.

2d Solution. — The interest for 4 yr. at 7½ per cent must be 4 times 7½, or 30 per cent, = $\frac{3}{10}$ of the principal. Hence,

	a = $537.48	= principal.
.3 of a = b =	161.244	= interest for 4 yr.
$\frac{1}{16}$ of b = c =	10.077	= interest for 3 mo.
⅛ of c = d =	1.679	= interest for 15 da.
b — c — d =	$149.488	= interest for 3 yr. 8 mo. 15 da.

3. What is the interest of $279.64 for 6 yr. 5 mo. 22 da. at ¼ per cent?

Solution. — The interest for 4 years at ¼ per cent must be 4 times ¼, or 1 per cent of the principal. Hence,

	a = 279.64	= principal.
.01 of a = b =	2.796	= int. for 4 yr.
½ of b = c =	1.398	= int. for 2 yr.
* ⅛ of b = d =	.310	= int. for 5⅓ mo., or 5 mo. 10 da.
† $\frac{1}{60}$ of c = e =	.023	= int. for 12 da.
	$4.527	= int. for 6 yr. 5 mo. 22 da.

* Since 5⅓ mo. = ⅓ of 4 yr., or 48 mo.

† Since 12 da. = ⅙ of 2 mo., and 2 mo. = $\frac{1}{12}$ of 2 yr., 12 days must equal ⅙ of $\frac{1}{12}$, or $\frac{1}{60}$ of 2 yr.

4. What is the interest of $537.47 for 1 yr. 10 mo. 15 da. at 3½ per cent?

Solution. — The interest of any sum for 3 years at 3½ per cent per year will be 10 per cent, or $\frac{1}{10}$ of that sum. Hence,

a =	$537.47	= principal.
$\frac{1}{20}$ of a = b =	26.873	= int. for 1 yr. 6 mo.
$\frac{1}{4}$ of b = c =	6.718	= int. for 4 mo. 15 da.
b + c =	$33.591	= int. for 1 yr. 10 mo. 15 da.

2d Solution.

a =	$537.47	= principal.
$\frac{1}{20}$ of a = b =	26.873	= int. for 1 yr. 6 mo.
$\frac{1}{6}$ of b = c =	4.479	= int. for 3 mo.
$\frac{1}{2}$ of c = d =	2.239	= int. for 1 mo. 15 da.
b + c + d =	33.591	= int. for 1 yr. 10 mo. 15 da.

NOTE. — It will be seen that, by this method, we first get the interest for any convenient time, and then take such part or parts of this as will give the interest for the required time.

5. What is the interest of $483.79 for 3 yr. 3 mo. 18 da. at 4½ per cent?

6. What is the interest of $538.71 for 1 yr. 7 mo. 24 da. at 9 per cent?

7. What is the interest of $875.37 for 2 yr. 7 mo. 13 da. at 8½ per cent?

8. What is the interest of $63.29 for 5 yr. 11 mo. 10 da. at 8½ per cent?

Suggestion. — The interest for 6 years, at 8½ per cent, is 50 per cent, or ½ of the principal, and since 20 days = ⅓ of 2 months = $\frac{1}{18}$ of 1 year = $\frac{1}{108}$ of 6 years, the interest for 20 days must equal $\frac{1}{108}$ of the interest for 6 years.

9. What is the interest of $56.84 from March 5, 1850, to May 25, 1851, at 5 per cent?

10. What is the interest of $138.46 from July 1, 1848, to Aug. 26, 1852, at 2½ per cent?

11. What is the interest of $273.81 from Sept. 28, 1847, to Oct. 31, 1852, at 7 per cent?

12. What is the amount of $783.25 from Aug. 28, 1846, to Feb. 29, 1852, at 4½ per cent?

13. What is the amount of $57.84 from Jan. 19, 1849, to Dec. 29, 1852, at 6⅔ per cent?

14. What is the amount of $278.49, from Sept. 13, 1841, to Oct. 10, 1846, at 7½ per cent?

15. Jan. 17, 1850, I borrowed 837 dollars, agreeing to pay interest at the rate of 6 per cent per year, and immediately put it on interest at the rate of 7½ per cent. Aug. 27, 1852, I collected the amount due to me, and paid that which I owed. How much did I gain by the transaction?

Suggestion. — Since I paid 6 per cent and received 7½ per cent interest on the sum I borrowed, my gain must have been 1½ per cent per year.

16. A speculator borrowed $2000, agreeing to pay interest at the rate of 9 per cent per year, and invested the money in land at $80 per acre. 3 mo. afterwards, he sold ¼ the land for $900, and the rest at $100 per acre, and expended the proceeds for flour. 2 mo. 15 da. afterwards he sold ⅛ of the flour for $600, and the remainder for what he paid for the whole, and immediately paid the amount of the borrowed money. How much was his gain?

17. A merchant, wishing to purchase 9 acres of land at $378.43 per acre, borrowed money for the purpose at the rate of 5 per cent. At the end of 3 yr. 9 mo. 15 da. he sold the land, receiving $400 per acre for ⅓ of it, and $475.28 for the remainder. Did he gain or lose, and how much?

18. Bought 397 yards of cloth at $3.75 per yard, payable in 6 months, with interest at 7½ per cent per year, and immediately sold it for $4 per yard, payable in 6 months, without interest. When the 6 months had elapsed, I collected the money due me, and paid my debt. Did I gain or lose, and how much?

19. Bought 397 yards of cloth at $4 dollars per yard, payable in 6 months, and immediately sold it at $3.75, cash, and put the money on interest at the rate of 7½ per cent. At the end

of 6 months I called in the money I had lent, and paid that which I owed. Did I gain or lose by the transaction, and how much?

20. Jan. 1, 1852, I borrowed $954, agreeing to pay interest at the rate of 5 per cent, and immediately expended it for cloth at $3 per yard. 4 days afterwards, I sold the cloth at $3.50 per yard, to be paid June 17, 1852. On receipt of the money, I immediately expended it for cloth at $1 per yard. July 1, 1852, I sold the cloth at $1.12½ per yd., payable Sept. 22, 1852. As soon as this debt was paid, I put the money on interest at 6 per cent. Jan. 1, 1853, I collected the amount due me, and paid that which I owed. How much had I gained by the transactions?

§ 17. NOTES ON TIME. BANK DISCOUNT.

If a person gives another a note promising to pay him a certain sum at the end of a given number of days, the note is not regarded as actually payable till three days after the time mentioned has elapsed. Thus, a note payable in 30 days cannot be collected till 33 days from the time it was given. The three days thus added are called *days of grace.*

When money is borrowed at a bank, it is usually made payable at the end of a certain number of days, and its interest for that number of days and three days more is deducted at the time it is borrowed. For instance, if a person should pass his note for $500 payable in 30 days at a bank, he would receive on it $500 minus its interest for 33 days, for which at the end of the 30 days and grace, he must pay $500. By this arrangement he pays interest on more money than he receives, as in the above example, where he pays interest on $500, while he has the use of but $497.25. Bank interest is called *discount* because it is thus deducted from the sum nominally borrowed, or from the face of the note, and the note on which the money is received is said to be *discounted.*

1. A note of $1200, payable in 60 days, was discounted at a bank at 6 per cent. How much was received on it?

Solution. — The interest of $1200 for 63 days, being 2 dimes

per day, is $12.60, which, deducted from $1200, leaves $1187.40 as the sum received.

2. How much would be received at a bank on a note of $200, payable in 90 days? $ 196.90

3. How much would be received at a bank on a note of $360, payable in 30 days? $ 355.02

4. I got my note for $1000, payable in 90 days, discounted at a bank, and immediately put the money received on it at interest. When the note became due, I collected the amount of what I had put on interest, and paid my note at the bank. How much did I lose by the transaction? How does the sum lost compare with the *interest* of the *bank discount* for the given time? $ .24

5. My note for $1000, payable in 6 months, was discounted at a bank, and I immediately put the money received on it at interest. When the note became due, I collected the sum due me, and paid that which I owed at the bank. How much did I lose by the transaction? $ 0.24

6. I had my note for $500, payable in 2 months, discounted at a bank, and immediately put the money on interest. When the note became due, I renewed it for the same time as before; and when the new note became due, I collected the amount due me, and paid my note at the bank. How much did I lose? .15

Suggestions. — It is evident that on the first note I lost 4 months and 6 days' interest on the bank discount, and that on the second I lost 2 months and 3 days' interest on the same discount. Therefore the sum of these losses, which is equivalent to the interest of the bank discount for 6 mo. 9 da., is the amount lost.

Again: since I paid nothing at the bank, except the bank discount at the time of renewing the note, and the second note when it became due, the actual value, at the time of settlement, of the sums paid, will be the amount of the bank discount for 2 mo. 3 da., plus the face of the note. The sum received will be the amount for 4 mo. 6 da. of the money obtained at the bank on the first note. The difference between the values paid and received is the loss.

7. I had my note for $600, payable in 4 months, discounted at a bank, and immediately lent the money received on it for just 1 year. When my note at the bank became due, I renewed it for the same time as before, and when this new note became due, I renewed it for such time that it became due at the end of the year, when I collected the amount of the sum I had lent, and paid my note at the bank. How much did I lose by the transactions? $1.44

8. July 1, 1851, I got my note for $1000, payable in 3 months, discounted at a bank, and immediately invested the money received on it, in land. Oct. 7, 1851, I sold the land at an advance of 12 per cent, receiving $\frac{1}{2}$ of the sales in cash, and a note for the other half, payable July 1, 1852, without grace, and to be on interest at 7 per cent after Jan. 1, 1852. I lent the cash at 6 per cent interest. When my note at the bank became due, I renewed it for the same time as before, and at the proper time renewed it again; and when this last note became due, I renewed it for such time that the new note would become due July 1, 1852. Allowing that I paid 6 per cent interest on the money borrowed at the bank, and that I made a complete settlement July 1, 1852, what was the amount of my gains? $100.36

§ 18. ENGLISH METHOD OF COMPUTING INTEREST.

In England, time is reckoned in years and days, but never in months. The year is regarded as 365 days. Interest is usually computed by first finding it for the years, and then for the days. In computing it for the days, it is well to notice that 73 days = $\frac{1}{5}$ of 1 year, that 5 days = $\frac{1}{73}$ of 1 year, and that 1 day = $\frac{1}{365}$ of 1 year.

When any part of the principal is expressed in shillings, pence, and farthings, it should be reduced to the decimal of a pound.*

* To do this, observe that each shilling is $\frac{1}{20}$, or .05 of £1, and that each farthing is $\frac{1}{960}$, or .001$\frac{1}{4}$ of £1. We shall then have as many times .05 of £1 as there are shillings, plus as many times .001$\frac{1}{4}$ of £1 as there are farthings

1. What is the interest of £327 17s. 7d. from May 7, 1851, to Sept. 4, 1852, at 5 per cent?

Solution. — From May 7, 1851, to May 7, 1852, is 1 year. There are 24 days left in May, to which adding the 30 in June, the 31 in July, the 31 in August, and the 4 in September, gives 120 days. The time, then, is 1 yr. 120 da. The principal equals £327.879. Hence we have the following written work: —

a =	£327.879	= principal.
.05, or $\frac{1}{20}$ of a = b =	16.39395	= int. for 1 yr.
$\frac{1}{365}$ of b = c =	.044914	= int. for 1 da.
120 times c = d =	5.389680	= int. for 120 da.
b + d = e =	£21.78363	= int. for 1 yr. 120 da. ?

Or we may have the following: —

a =	£327.879	= principal.
.05, or $\frac{1}{20}$ of a = b =	16.39395	= int. for 1 yr.
$\frac{1}{73}$ of b = c =	.224574	= int. for 5 da.
23 times c = d =	5.165202	= int. for 115 da.
b + c + d = e =	£21.783726	= int. for 1 yr. 120 da. ?

Calling this £21.784, we have £21 15s. 8d. 1 qr. as the answer. The multiplications required in solving these examples render it necessary to carry out the work to places below thousandths, though we do not care to have them appear in the answer.

2. What is the interest of £47 9s. 4d. 1 qr. from May 17, 1849, to Aug. 23, 1852, at 5 per cent?

---

in the pence and farthings. But as all values less than $\frac{1}{2}$ of .001 of £1 are so small that they may be disregarded, the result will be sufficiently accurate for ordinary purposes, if we regard each farthing as .001 of £1, observing to add .001 if there are more than 12 and less than 36 farthings, and .002 if there are more than 36. By adding this result to the value of the shillings, we shall have the decimal expression required. For example: To find what part of £1 is equal to 9s. 8d. 1 qr., we have 9s. = 9 times £.05 = £.45; 8d. 1 qr. = 33 qr. = £.033 + £.001 = £.034. Therefore 9s. 8d. 1 qr. = £.45 + £.034 = £.484. The reverse operation will get the value of the decimal expression, in terms of shillings, pence, and farthings.

3. What is the interest of £148 19s. 9d. 3 qr. from Oct. 23, 1850, to Nov. 11, 1852, at 5 per cent? £ 15. 5. 4.

4. What is the amount of £361 13s. 2d. 1 qr. from July 18, 1847, to April 12, 1850, at 5 per cent? £ 49. 4 3.

5. What is the amount of £248 18s. 10d. 3 qr. from Dec. 5, 1849, to March 3, 1852, at 4½ per cent?

6. What is the interest of £548 15s. 7d. 3 qr. from July 29, 1847, to March 12, 1850, at 2½ per cent? £ 55. 17 6

7. What is the amount of £258 19s. 5d. 2 qr. from Jan. 1, 1849, to Sept. 29, 1852, at 4 per cent? £ 39. 15 9

8. What is the amount of £329 7s. 1d. 3 qr. from Nov. 13, 1850, to Dec. 1, 1852, at 3 per cent?

9. What is the interest of £481 13s. 5d. 1 qr. from April 19, 1842, to May 3, 1847, at 5 per cent?

10. What is the amount of £222 2s. 2d. 2 qr. from Feb. 29, 1848, to Jan. 1, 1852, at 4 per cent?

§ 19. PARTIAL PAYMENTS.

The principle adopted by the Supreme Court of the United States, and by that of Massachusetts and most of the other states, as the one to be applied in determining the sum due on a promissory note or bond on which payments have been made at different times, is that as much of the payment as is necessary to pay the interest due at the time the payment is made should be appropriated to that purpose, and the surplus to the payment of the principal. If, however, any payment is less than the interest at the time due, the principal remains unaltered, and on interest, as before, till some payment is made, which, with the preceding neglected payments, is more than sufficient to pay the interest; when we proceed as if a single payment, equal to the sum of the last payment and the preceding neglected ones, had been made.*

* The method adopted by the court of Connecticut differs from the above only in this respect — that if a payment greater than the interest at the time due be made before the principal has been on interest one year, the person

Example.

1. $750.00 Boston, April 7, 1848.

For value received, I promise to pay James Sullivan, or order, seven hundred and fifty dollars, on demand, with interest.

Edward Delano.

On this note are the following indorsements: —

Jan. 17, 1849. Received one hundred dollars.
March 13, 1850. Received twenty-five dollars.
Feb. 19, 1851. Received thirty dollars.
Aug. 3, 1851. Received two hundred dollars.
Jan. 1, 1852. Received one hundred and fifty dollars.

What was due on the note at the time of settlement, Aug. 14, 1852?

The following exhibits a good form of writing the work in such examples, and, in connection with the explanations following, will be a sufficient illustration of the process: —

	$750.00	= 1st principal, April 7, 1848.
April 7, 1848, to Jan. 17, 1849, = 9 mo. 10 da. = 280 da.	35.00	= int. 280 days at 1¼ dimes per day.
	$785.00	= amt. 9 mo. 10 da., or 280 da.
	100.00	= 1st payment.
	$685.00	= 2d principal, Jan. 17, 1849.
Jan. 17, 1849, to Aug. 3, 1851, = 2 yr. 6 mo. 17 da.	68.50	= int. 20 mo.
	34.25	= int. 10 mo.
	1.712	= int. 15 da.
	.228	= int. 2 da.
	$789.69	= amt. 2 yr. 6 mo. 17 da.

making it is allowed interest on it to the end of the year; that is, its amount from the time it was made to the end of the year, is deducted from the amount of the principal to the same time. If settlement be made before the principal has been on interest one year, interest is allowed on the payments from the time they were made to the time of settlement.

Amt. brought over,	$789.69	
	255.00	= 2d, 3d, and 4th payments.
	$534.69	= 3d principal, Aug. 3, 1851.
Aug. 3, 1851, to Jan. 1, 1852, = 4 mo. 29 da.	13.367	= int. 5 mo.
	.089	= int. 1 da.
	$547.968	= amt. 4 mo. 29 da.
	150.00	= 5th payment.
	$397.968	= 4th principal, Jan. 1, 1852.
Jan. 1, 1852, to Aug. 14, 1852, = 7 mo. 13 da.	13.265	= int. 6 mo. 20 da.
	1.326	= int. 20 da.
	.198	= int. 3 da.
	$412.757	= amt. due Aug. 14, 1852, = Ans.

Explanation. — As it is obvious that the first payment was greater than the interest at the time due, we get the amount of the note to that time, and deduct from it the payment. The remainder is a new principal. The second payment, $25, is evidently less than the interest then due; for the time is over 1 year, while the principal, between $600 and $700, gives more than $36 interest per year. Similar considerations will at once show that the interest to the time of the third payment must be greater than the second and third payments together. But the fourth payment, together with the second and third, is very evidently more than sufficient to pay the interest then due; therefore we get the amount of the new principal to the time of the fourth payment, and subtract from it the sum of the second, third, and fourth payments, thus getting our third principal. As it is evident that the interest of this principal to the time of the fifth payment is less than that payment, we find its amount, and from it subtract the payment. This gives us the fourth principal, which is on interest till the time of settlement, and hence its amount is the sum due.

2. $850.00 Boston, April 7, 1847.

For value received, I promise to pay Albert Simmons, or order, eight hundred and fifty dollars, on demand, with interest.

Isaac Goodrich.

On this note were the following indorsements: —

June 19, 1848. Received one hundred and twenty-five dollars.
Jan. 7, 1849. Received eighty-three dollars.
Sept. 27, 1849. Received one hundred dollars.
May 1, 1850. Received twenty dollars.
Aug. 28, 1850. Received two hundred dollars.
Jan. 1, 1851. Received one hundred dollars.

How much was due on the note, Oct. 13, 1851?

3. $1000.00 Providence, Nov. 28, 1848.

I promise to pay Bradford Allen, or order, one thousand dollars, on demand, with interest. Value received.

Henry Williams.

On this note were the following indorsements: —

July 23, 1849. Received eighty dollars.
Feb. 28, 1850. Received fifteen dollars.
June 27, 1850. Received twenty dollars.
April 2, 1851. Received twenty-five dollars.
Dec. 20, 1851. Received five hundred dollars.
May 17, 1852. Received three hundred dollars.

This note was settled Aug. 14, 1852. How much was then due? $ 254.47

4. 645\frac{7}{100}$ Worcester, Dec. 20, 1846.

For value received, we promise to pay Alfred Lincoln, or order, six hundred and forty-five dollars and 75 cents, on demand, with interest.

Thompson & French.

Indorsements.

Nov. 8, 1848. Received forty dollars.
April 16, 1849. Received three hundred dollars.
March 10, 1851. Received two hundred and fifty dollars.
Sept. 8, 1851. Received sixty dollars.
The note was settled Jan. 1, 1852. How much was then due?

5. $1275.00. Bridgewater, Sept. 29, 1845.

For value received, we promise to pay Lincoln and Wood twelve hundred and seventy-five dollars, on demand, with interest.

Paine, Root, & Co.

Indorsements.

Aug. 5, 1846. Received three hundred dollars.
Sept. 22, 1847. Received four hundred dollars.
May 25, 1848. Received two hundred dollars.
June 17, 1849. Received one hundred and fifty dollars.
Nov. 13, 1850. Received one hundred and fifty dollars.

The balance was paid March 1, 1851. How much was then due?

6. $3000.00 Lowell, April 3, 1849.

For value received, I promise to pay James Wyman, or order, three thousand dollars, on demand, with interest.

Edward Robinson.

Indorsements.

Nov. 1, 1849. Received five hundred dollars.
Dec. 27, 1850. Received ninety dollars.
March 25, 1851. Received fifty dollars.
July 18, 1851. Received six hundred dollars.
Sept. 13, 1851. Received one thousand dollars.
Jan. 1, 1852. Received one thousand dollars.
The note was settled Nov. 8, 1852. How much was then due?

§ 20. MERCHANTS' METHOD WHEN DEBTS ARE PAID WITHIN A YEAR.

When notes or debts of any kind, on which partial payments have been made, are paid in full within one year from the time interest commences, merchants often determine the sum to be paid on settlement, as they would if nothing is due on a note till it is paid in full; that is, they find the amount of the note to the time of settlement, and the amount of each payment from the time it was made till the time of settlement, and then consider the excess of the amount of the note over the sum of the amounts of the several payments to be the sum due on settlement.

1. $500.00 Worcester, July 8, 1851.

For value received, I promise to pay John F. Barnard, or order, five hundred dollars, on demand, with interest.

William H. West.

Indorsements.

Sept. 23, 1851. Received sixty dollars.
Nov. 20, 1851. Received one hundred dollars.
Jan. 17, 1852. Received two hundred dollars.
Feb. 8, 1852. Received fifty dollars.

How much was due May 11, 1852?

Solution.

$500.00 = principal.
25.25 = int. 10 mo. 3 da.
$525.25 = amt. of note to May 11, 1852.

$60.00 = 1st payment, Sept. 23, 1851.
2.28 = int. 7 mo. 18 da.
100.00 = 2d payment, Nov. 20, 1851
2.85 = int. 5 mo. 21 da.
200.00 = 3d payment, Jan. 17, 1852.
3.80 = int. 3 mo. 24 da.
50.00 = 4th payment, Feb. 8, 1852.
.775 = int. 3 mo. 3 da.

$419.705 = amt. of payments, May 11, 1852.
$106.55 = balance due May 11, 1852 = Ans.

2. $728.00 Springfield, Sept. 7, 1849.

For value received, I promise to pay A. Parish, or order, seven hundred and twenty-eight dollars, on demand, with interest

William Mitchell.

Indorsements.

Oct. 3, 1849. Received eighty dollars.
Dec. 1, 1849. Received ninety dollars.
Feb. 4, 1850. Received one hundred dollars.
March 2, 1850. Received forty dollars.
June 1, 1850. Received eighty dollars.

How much was due Aug. 2, 1850? 364.15

3. 583\frac{74}{100}$ Lowell, Jan. 18, 1850.

For value received, I promise to pay C. C. Chase, or order, five hundred and eighty-three dollars and seventy-five cents, on demand, with interest. A. H. Fiske.

Indorsements.

March 5, 1850. Received fifty dollars.
April 1, 1850. Received seventy-five dollars.
June 17, 1850. Received twenty-eight dollars.
July 3, 1850. Received one hundred dollars.
Oct. 2, 1850. Received ninety dollars.
Dec. 27, 1850. Received seventy dollars.

How much was due Jan. 7, 1851?

§ 21. TO FIND THE TIME.

The methods illustrated in the following solutions will enable us to find the time, when we know the principal, interest, and rate.

1. How long must $420 be on interest at 6 per cent to gain $32.27?

Solution. — The interest of $420 for 6 days is $.42. If it takes 6 days to gain $.42, it will take $\frac{3227}{42}$ of 6 days to gain $32.27.

$$\frac{3227}{42} \text{ of } 6 \text{ days} = \frac{\overset{461}{\cancel{3227}} \times \cancel{6}}{\underset{7}{\cancel{42}}} \text{ days} = 461 \text{ days} = 15 \text{ mo. } 11 \text{ days.}$$

NOTE. — The reduction was made by cancelling 6 from 6 and 42, and 7 from 7 and 3227.

2. How long must $357 be on interest at 6 per cent to gain $29.869?

Solution. — The interest of $357 for 6 days is $.357. If it takes 6 days to gain $.357, it will take $\frac{29869}{357}$ of 6 days to gain $29.869.

$$\frac{29869}{357} \text{ of } 6 \text{ da.} = \frac{\overset{251}{\cancel{29869}} \times \overset{2}{\cancel{6}}}{\underset{119}{\cancel{357}}} \text{ da.} = 502 \text{ da.} = 16 \text{ mo. } 22 \text{ da.}$$

NOTE. — The reduction was made by cancelling 3 from 6 and 357, and 119 from 119 and 29869.

3. How long must $136.80 be on interest at 7 per cent to gain $2.793.

Solution. — The interest of $136.80 for 1 year, or 360 days, at 7 per cent, is $9.576. If it takes 360 days to gain $9.576, it will take $\frac{2793}{9576}$ of 360 days to gain $2.793.

$$\frac{2793}{9576} \text{ of } 360 \text{ da.} = \frac{\overset{21}{\cancel{2793}} \times \overset{5}{\cancel{360}}}{\underset{133}{\cancel{9576}}} \text{ da.} = 105 \text{ da.} = 3 \text{ mo. } 15 \text{ da.}$$

NOTE. — The reduction was made by cancelling 72 from 360 and 9576, and 133 from 133 and 2793.

By this method of solution, we first select some convenient time for which to compute the interest. Then the required time will be the same part of the time selected that the given interest is of the interest for the selected time. The selected time should be one for which the interest can be easily

computed, as, when the rate is 6 per cent per year, 6 da., 60 da = 2 mo., 600 da. = 20 mo., 6000 da. = 200 mo.; and when the rate is other than 6 per cent, 1 year = 360 da., or such part of 1 year as shall give for the interest 1 per cent, or some other equally convenient part of the principal.

When there is a fractional part of a day in the result, the fraction may be omitted if it be less than ½, but if it be more than ½, 1 may be added to the number of days.

4. How long must $427.32 be on interest at 6 per cent to gain $19.68?

5. How long must $186.75 be on interest at 6 per cent to gain $12.45?

6. How long must $576.00 be on interest at 6 per cent to gain $65.568?

7. How long must $528.32 be on interest at 6 per cent to gain $86.38? 2 - 5 - 2

8. How long must $687.35 be on interest at 6 per cent to gain $132.63? 3 - 2 - 16

9. How long must $378.50 be on interest at 7 per cent to gain $4.542?

10. How long must $56.34 be on interest at 8 per cent to gain $18.78?

11. How long must $873.70 be on interest at 5 per cent to gain $17.474? 4 - 24

12. How long must $594.00 be on interest at 4 per cent to gain $60.654? 2 - 6 - 19

§ 22. EQUATION OF PAYMENTS.

When one man owes another sums of money payable at different times, it may be desirable to determine when the whole can be paid without gain or loss to either party. The process of doing this is called *Equation of Payments*, and the time sought is called the *Equated Time*. It is obvious that if a debt be not paid till after it has become due, the debtor gains the use of it from the time it became due to the time of payment; while if it be paid before it becomes due, the debtor loses the use of

the sum paid from the time of payment to the time when it would justly have been due. The use of any sum of money is regarded as worth its interest for the time it is used. The application of the foregoing principles is illustrated in the following problems and solutions: —

1. Mr. Lincoln owes Mr. Wood $400, due in 5 months, $600, due in 9 months, and $200, due in 12 months. When can the whole be paid without gain or loss to either party?

Solution. — By the conditions of the question, Mr. Lincoln is entitled to the use or

Interest of	$400	for 5 mo.	= $10.00
Interest of	$600	for 9 mo.	= $27.00
Interest of	$200	for 12 mo.	= $12.00
or to use	$1200	till its int.	= $49.00

The interest of $1200 being $6 per month, he is entitled to keep it as many months as there are times $6 in $49, which are $8\frac{1}{6}$ times. Therefore he is entitled to keep it $8\frac{1}{6}$ months, or 8 months and 5 days.

1*st Proof.* — By paying the whole at the equated time, Mr. Lincoln gains the use of the first debt from the time it was due to the equated time, and loses that of the second and third from the equated time to the time when they would otherwise have been due. That is, he

gains interest of	$400.00	for 3 mo.	5 da. =	$6.33\frac{1}{3}$
and loses interest of	$600.00	for	25 da. =	$2.50
and loses interest of	$200.00	for 3 mo.	25 da. =	$3.83\frac{1}{3}$

sum of losses = $6.33\frac{1}{3}$ = the gain, which shows the work to be correct.

2*d Proof.* — If each debt should be paid when it becomes due, Mr. Wood will, when the last debt is paid, have had the use of $400 for 7 mo. and of $600 for 3 mo., which at 6 per cent is equivalent to $14 + $9 = $23 interest. If, however,

the sum of the debts should be paid at the equated time, Mr. Wood would, at the end of 12 months, when the last debt would otherwise have been paid, have had the use of $1200 for 3 mo. 25 da., which, at 6 per cent, is worth $23 interest. This shows that he would have the same interest in one case as in the other, and thus proves the first result correct.

2d Solution. — Another solution similar in character to the last can be obtained by ascertaining how much would be gained or lost by paying the entire debt at any assumed time, and from that getting the equated time.

For instance, suppose that the entire debt had been paid at the end of 9 months. Then Mr. Lincoln would have

gained interest on $400 for 4 months = $8.00
and lost interest on $200 for 3 months = $3.00
equivalent to a gain of $5.00

which shows that 9 months is as many days longer than the true time as it will take for $1200 to gain $5 at interest. We find (by § 21) that it will take $1200, at 6 per cent interest, 25 days to gain $5. Therefore the equated time = 9 mo. — 25 da. = 8 mo. 5 da.

3d Solution. — When the numbers are convenient, as in this example, a method like the following can be used to advantage: —

The sum of the debts is $1200, of which the first debt is $\frac{1}{4}$, the second $\frac{1}{2}$, and the third $\frac{1}{8}$. But the use of $\frac{1}{8}$ of a sum 5 mo. is worth as much as the use of the whole of it for $\frac{1}{4}$ of 5 mo., or $1\frac{2}{3}$ mo.; the use of $\frac{1}{2}$ of a sum for 9 mo. is worth as much as the use of the whole of it for $\frac{1}{2}$ of 9 mo., or $4\frac{1}{2}$ mo.; and the use of $\frac{1}{8}$ of a sum for 12 mo. is worth as much as the use of the whole of it for $\frac{1}{8}$ of 12 mo., or 2 mo. Therefore Mr. Lincoln is entitled to the use of the sum of the debts for $1\frac{2}{3}$ mo. + $4\frac{1}{2}$ mo. + 2 mo. = $8\frac{1}{3}$ mo. = 8 mo. 5 da.

4th Solution. — The following method is much used, but we think the method by interest will ordinarily be found preferable: —

The use of $400 for 5 mo. is worth as much as the use of $1

for 400 times 5 mo., or 2000 mo. The use of \$600 for 9 mo. is worth as much as the use of \$1 for 600 times 9 mo., or 5400 mo. The use of \$200 for 12 mo. is worth as much as the use of \$1 for 200 times 12 mo., or 2400 mo. Therefore Mr. Lincoln is entitled to the use of the entire debt for such time as will be equivalent to the use of \$1 for 2000 mo. + 5400 mo. + 2400 mo., or 9800 mo. But as the use of \$1 for 9800 mo. is equivalent to the use of \$1200 for $\frac{1}{1200}$ of 9800 mo., or $8\frac{1}{6}$ mo., he can keep the entire debt $8\frac{1}{6}$ mo., or 8 mo. 5 da.

NOTES. — 1st. As the equated time will be the same, whatever be the rate of interest, the rate may always be considered to be that which is most easily calculated.

2d. The equated time will frequently contain a fraction of a day; but if the fraction be less than $\frac{1}{2}$, it may be disregarded, or if it be more than $\frac{1}{2}$, 1 may be added to the number of days.

2. A owes B \$250, due in 3 mo., \$400, due in 6 mo., and \$350 due in 8 mo. What is the equated time of payment?

3. I owe \$700, payable as follows: \$150 in 3 mo., \$184 in 7 mo., and the rest in 11 mo. When can I pay the whole without gain or loss?

4. I owe \$960, payable as follows: \$180 in 4 mo. 20 da., \$348 in 6 mo. 15 da., \$234 in 8 mo. 5 da., and the rest in 10 mo. 13 da. Required the equated time of payment.

5. A trader bought \$1800 worth of goods, agreeing to pay $\frac{1}{3}$ of the money down, $\frac{1}{6}$ of it in 5 mo., $\frac{1}{3}$ of it in 6 mo., $\frac{1}{3}$ of it in 9 mo., and the rest in 12 mo. At what time may the whole be paid?

6. A owes B \$800, payable in 10 mo.; but to accommodate B, he pays \$250 down. When ought the remainder to be paid?

Suggestion. — Since $\$250 = \frac{250}{800} = \frac{5}{16}$ of \$800, the problem is equivalent to this: If A pays $\frac{5}{16}$ of the debt down, how much longer than 10 months ought he to keep the remaining $\frac{11}{16}$ of it? The use of $\frac{5}{16}$ of it for 10 mo. is worth as much as the use of $\frac{1}{16}$ of it for 5 times 10 mo., or 50 mo., and the use of $\frac{1}{16}$ of it for 50 mo. is worth as much as the use of $\frac{11}{16}$ of it for $\frac{1}{11}$

of 50 mo., or $4\frac{6}{11}$ mo., which, added to 10 mo., gives $14\frac{6}{11}$ mo. as the time during which A is entitled to keep the remaining part of his debt.

7. I owe $1000, payable in 9 mo.; but to accommodate my creditor, I pay $300 down, and agree to pay $300 more in 2 mo. How long ought I, in justice, to keep the remainder?

8. I owe $600, payable in 8 mo. 15 da., and $400, payable in 12 mo.; but afterwards agree to pay $400 down, and $300 in 2 mo. 20 da., on condition that I may keep the remainder enough longer to compensate for my loss. When will the remainder become due?

9. A owes B $480, due in 1 yr., and B owes A $720, due in 1 yr. 6 mo. If A should pay his debt at once, when ought B to pay his?

10. A owes B $144, due in 6 mo. 20 da., and B. owes A $324, due in 1 yr. 4 mo. 20 da. If A should pay half of his debt now, and the other half when, by the conditions, the whole debt was due, when ought B to pay the whole of his?

11. A owes B $600, due in 9 mo., and B owes A $900, due in 15 mo. If A does not pay his debt till B's would otherwise have become due, when ought B, in justice, to pay his debt to A?

12. Bought a lot of goods, for which I agreed to pay $437.75 in 3 mo., $394.25 in 6 mo., and $628.19 in 8 mo. When may the whole be paid without gain or loss?

§ 23. TO FIND DATE OF EQUATED TIME.

The best method of solving such examples as the following is to see how much interest will be gained or lost by paying the sum of the debts at any assumed time. It will be well, as a general thing, to select for the assumed time a date on which one of the debts becomes due, as by that means we shall avoid the necessity of reckoning interest on that debt. Reference should also be had to the probable equated time. The time is reckoned by counting the days between the dates considered, as in the English method of computing interest.

1. James Brown owes William Greene the following debts, viz.: $534.83, due Jan. 7, 1852; $285, due April 4, 1852; $327.38, due July 3, 1852; and $438.75, due Aug. 17, 1852. When may the whole be paid without gain or loss?

Solution. — Suppose that April 4, 1852, be selected as the assumed time. Then Mr. Brown would

gain int. on $534.83 from Jan. 7 to April 4, 88 da. = $7.84
no int. on $285.00 due at assumed time.
lose int. on $327.38 from April 4 to July 3, 90 da. = $4.91
lose int. on $438.75 from Apr. 4 to Aug. 17, 135 da. = $9.87

Sum of debts = $1585.96 Sum of losses = $14.78
Excess of losses over gains $6.94

Showing that Mr. Brown is entitled to keep $1585.96, the entire debt due, as many days after April 4 as it will take it to gain $6.94 interest. This, found by § 21, is 26 days, plus a fraction less than $\frac{1}{2}$.

Therefore the equated time is 26 days after April 4, which is April 30.

Again: suppose that July 3 be selected as the assumed time. Then Mr. Brown would

gain int. on $534.83 from Jan. 7 to July 3, 178 da., $15.86
gain int. on $285.00 from April 4 to July 3, 90 da., 4.27
Sum of gains $20.13
no int. on $327.38 due at assumed time.
lose int. on $438.75 from July 3 to Aug. 17, 45 da., 3.29

Sum of debts = $1585.96 Excess of gain over loss, $16.84

Showing that Mr. Brown ought to pay $1585.96, the entire debt due, as many days before July 3 as it will take it to gain $16.84 interest. This, found as before, is 64 days nearly. Therefore the equated time is 64 days before July 3, which is April 30, as before.

Proof. — By paying the debt on April 30, Mr. Brown will

gain int. of $534.83 from Jan. 7 to April 30, 114 da.	$10.16	
gain int. of $285.00 from Apr. 4 to Apr. 30, 26 da.	1.23	
Sum of gains, ...		$11.39
lose int. of $327.38 from April 30 to July 3, 64 da.	$3.49	
lose int. of $438.75 from April 30 to Aug. 17, 109 da.	$7.97	
Sum of losses, ...		$11.46
Excess of loss over gain, ...		$00.07

which, being less than the interest of $1585.96 for a half day, shows that April 30 is the correct equated time.

2. I owe $387.53, due Nov. 7, 1851; $467.81, due Dec. 21, 1851; $256.19, due Feb. 11, 1852; $136.43, due Mar. 1, 1852; and $387.59, due May 3, 1852. What is the equated time of payment?

3. I owe $2867, due April 15, 1850; $1642, due July 27, 1850; $4371, due Oct. 8, 1850; and $5940, due Jan. 1, 1851. What is the equated time of payment?

4. I owe $628.18, due Dec. 17, 1852; $427.19, due Dec. 23, 1852; $371.16, due Dec. 30, 1852; $587.83, due Jan. 3, 1853; $987.62, due Jan. 7, 1853; and $843.28, due Jan. 14, 1853. What is the equated time of payment?

5. I owe $543.28, due April 24, 1853; $723.13, due May 11, 1853; $484, due Sept. 3, 1853; $426.18, due Oct. 10, 1853; $236, due Nov. 10, 1853. What is the equated time of payment?

6. What is the equated time for paying the following debts: $600, due March 7, 1850; $400, due June 11, 1850; $800, due Aug. 17, 1850; $500, due Oct. 3, 1850; and $1000, due Nov. 27, 1850?

§ 24. EQUATION OF ACCOUNTS.

The method of finding the equated time when each party owes the other, that is, when there are entries on both the *debit* and *credit* side of an account, does not differ in principle from that in which there are entries only on one side. The following example and solution will illustrate it: —

1. The account books of A and B show that

A owes B	And that B owes A
$426.70, due Jan. 7, 1852.	$148.37, due Dec. 23, 1851.
$413.65, due Feb. 3, 1852.	$173.19, due Jan. 30, 1852.
$169.28, due Apr. 13, 1852.	$587.23, due May 7, 1852.
$328.57, due Aug. 29, 1852.	$658.45, due Sept. 30, 1852.

When ought the balance to be paid?

Solution. — Suppose that April 13, 1852, be the assumed time of payment. Then, on debts due from A to B, A will

gain int. of $426.70, Jan. 7 to Apr. 13, 1852, 97 da. = $6.90
gain int. of $413.65, Feb. 3 to Apr. 13, 1852, 70 da. = $4.83
nothing on $169.28, due Apr. 13, 1852.

showing the sum of gains to be		$11.73
And he will lose interest of $328.57, April 13 to Aug. 29, 1852, 138 da. = ...		7.56
Excess of gain over loss, ...		$4.17

On debts due from B to A, A will

lose int. of $148.37 from Dec. 23, 1851, to April 13, 1852, 112 da. = ...	$2.76	
lose int. of $173.19 from Jan. 30, 1852, to April 13, 1852, 74 da. = ...	2.14	
Sum of losses, ...		$4.90
And he will gain int. of $587.23 from April 13 to May 7, 24 da. = ...	$ 2.35	
and he will gain int. of $658.45 from April 13 to Sept. 30, 170 da. = ...	18.65	
Sum of gains, ...		$21.00
Excess of gain over loss, ...		$16.10
which, added to first gain, gives for the entire gain of A, or loss of B, ...		$20.27

But if A should pay B $1338.20, the sum of the debts due him, and B should pay A $1567.24, the sum of the debts due him, B would pay A $229.04 more than A would pay B. The

problem now resolves itself into this: If by B's paying A $229.04, April 13, 1852, A gains and B loses $20.27 interest, when can he pay it without any gain or loss of interest? The answer evidently is, As many days after April 13, 1852, as it will take $229.04, or, disregarding the cents, $229, to gain $20.27 interest. This, found by methods before explained, is 531 days = 1 yr.* 166 da., and shows the equated time to be Sept. 26, 1853, which may be proved as were the former examples.

The following form of writing the work is more convenient than the preceding: —

On debts due from A to B, A gains		
int. of $426.70 from Jan. 7 to Apr. 13, 1852, 97 da. =	$ 6.90	
int. of $413.65 from Feb. 3 to Apr. 13, 1852, 70 da. =	$ 4.83	
And on debts due from B to A, A gains		
int. of $587.23 from April 13 to May 7, 1852, 24 da. =	$ 2.35	
int. of $658.45 from April 13 to Sept. 30, 1852, 170 da. =	$18.65	
Total gain of A, or loss of B,		$32.73
On debts due from A to B, A loses		
int. of $328.57 from April 13 to Aug. 29, 1852, 138 da. =	$ 7.56	
And on debts due from B to A, A loses		
int. of $148.37 from Dec. 23, 1851, to April 13, 1852, 112 da. =	$ 2.76	
int. of $173.19 from Jan. 30, 1852, to April 13, 1852, 74 da. =	$ 2.14	
Total loss of A, or gain of B,		$12.46
Excess of A's gain over his loss,		$20.27

As B loses what A gains, he is entitled to keep $229.04, the balance due to A, till its interest shall be $20.27. This time, found as before, is Sept. 26, 1853.

* Reckoning the year as 365 days, as is always done in such cases, unless it includes February of leap year, when it is reckoned as 366 days.

NOTE. — Although accounts like the above are sometimes settled by notes payable at the equated time, they are more frequently settled by notes payable at some more convenient time, or by cash. In all such cases, allowance is made for the interest gained or lost. Thus, if the above account should be settled by cash April 13, 1852, $20.27 would be deducted from the balance due from B to A, in order to compensate B for the interest he would lose; that is, B would pay A $229.04 — $20.27 = $208.77. If it should be paid May 1, 1852, B would have to pay A $.69 (the interest of the balance due A from April 13 to May 1) more than if he had paid it April 13; or, which is the same thing, he would have to pay the balance $229.04, minus its interest $19.58, from May 1, 1852, to the equated time. If the balance due at any given time had been originally required, it should have been found directly by making the given time the "assumed time."

2. By the respective accounts of Henry Lane and William Pond, it appears that

Pond owes Lane	And that Lane owes Pond
$ 876.37, due April 5, 1852.	$ 228.13, due April 28, 1852.
$ 579.48, due May 3, 1852.	$ 347.16, due June 3, 1852.
$ 487.83, due June 11, 1852.	$ 313.27, due July 28, 1852.
$ 145.38, due Aug. 8, 1852.	$ 839.42, due Sept. 1, 1852.
$2089.06, = amt. due Lane.	$1727.98, = amt. due Pond.

$2089.06 — $1727.98 = $361.08 = balance due Lane.

When can this balance be paid without gain or loss to either party?

Solution. — Suppose it to be paid June 11, 1852. Then will Mr. Pond gain

interest of $876.37, April 5 to June 11, 67 da.	$9.79
interest of $579.48, May 3 to June 11, 39 da.	3.77
no int. on $487.83	
interest of $313.27, June 11 to July 28, 47 da.	2.45
interest of $839.42, June 11 to Sept. 1, 82 da.	11.47
Sum of gains = ...	$27.48

He will lose

interest of $145.38, June 11 to Aug. 8, 58 da.	$1.41
interest of $228.13, April 28 to June 11, 44 da.	1.67
interest of $347.16, June 3 to June 11, 8 da.	.46
Sum of losses, ...	$ 3.54
Excess of gain over loss, . .	$23 94

As Mr. Pond gains this interest on money which he owes, he ought to pay the debt ($361.08, the balance of the account) as many days before June 11, 1852, as it will take for it to gain $23.94 interest. This gives for the equated time 398 days before June 11, 1852, which is May 10, 1851. The sum necessary to settle the account after the equated time will be the amount of the balance, $361.08, from the equated time to the time of settlement.

3. When was the balance of the following account due?

Dr. George Ide, in account with James Snow. Cr.

1849.				1849.			
Jan. 17.	To Mdse.	$336	18	Feb. 1.	By Mdse.	$421	30
Jan. 31.	To Mdse.	443	17	Feb. 27.	By Mdse.	620	00
March 7.	To Cash,	218	63	Mar. 13.	By Mdse.	283	17
April 17.	To Mdse.	500	00	Apr. 29.	By Mdse.	482	29
May 28.	To Mdse.	84	36	June 1.	By Mdse.	825	13

4. When was the balance of the following account due?

Dr. George Black in account with John Brown. Cr.

1850.				1850.			
May 13.	To Mdse. 4 mo.	$431	17	June 1.	By Mdse. 3 mo.	$223	62
July 25.	To Mdse. 3 mo.	256	38	July 7.	By Mdse. 6 mo.	150	00
Aug. 8.	To Mdse. 6 mo.	431	72	July 22.	By Mdse. 3 mo.	250	00
Sept. 23.	To Mdse. 3 mo.	585	41	Sept. 1.	By Cash, ...	300	00
Nov. 7.	To Mdse. 3 mo.	738	16	Nov. 23.	By Mdse. 2 mo.	138	16
				Dec. 1.	By Mdse. 3 mo.	122	31

NOTE. — 4 mo., 3 mo., &c., means that goods were sold at so many months credit.

5. When was the balance of the following account due?

Dr. Ira Doe in account with Jesse Buck. Cr.

1845.				1845.			
July 7.	To Mdse. 6 mo.	$ 150	75	Aug. 17.	By Mdse. 5 mo.	$ 500	00
Sept. 3.	To Mdse. 6 mo.	233	16	Sept. 20.	By Mdse. 6 mo.	1000	00
Sept. 5.	To Mdse. 4 mo.	1463	25	Oct. 19.	By Mdse. 4 mo.	1423	87
Sept. 17.	To Mdse. 3 mo.	857	32	Nov. 10.	By Mdse. 2 mo.	468	37
Sept. 23.	To Mdse. 6 mo.	4261	35	Dec. 20.	By Mdse. 3 mo.	3942	41
Oct. 14.	To Mdse. 6 mo.	2429	28	Dec. 25.	By Cash	2000	00
Oct. 31.	To Mdse. 6 mo.	138	60				
Nov. 20.	To Mdse. 4 mo.	2000	00				
Dec. 8.	To Mdse. 3 mo.	1500	00				

6. What was due on the following account Jan. 1, 1853?

Dr. George Mann in account with Henry Guild. Cr.

1852.				1853.			
May 5.	To Bal. 3 mo.	$513	43	Apr. 20.	By Mdse. 6 mo.	$328	13
June 27.	To Mdse. 4 mo.	624	27	May 10.	By Mdse. 3 mo.	143	27
July 3.	To Mdse. 6 mo.	831	18	June 13.	By Mdse. 4 mo.	837	19
Sept. 5.	To Mdse. 2 mo.	47	62	July 7.	By Mdse. 6 mo.	56	18
Sept. 17.	To Mdse. 3 mo.	125	53	Aug. 20.	By Mdse. 4 mo.	123	42
Oct. 19.	To Cash, . . .	387	00	Oct. 1.	By Mdse. 3 mo.	78	36
Dec. 1.	To Cash, . . .	629	28	Nov. 23.	By Mdse. 2 mo.	127	14

7. What was due on the following account, Jan. 1, 1853, interest being 7 per cent, and 4 mo. credit being allowed on each entry?

Dr. David H. Daniels in account with George W. Dean. Cr.

1852.				1852.			
July 8.	To Mdse. . .	$ 236	17	July 3.	By Mdse.	$439	27
Aug. 1.	To Sundries,	819	63	July 25.	By Mdse.	213	16
Sept. 4.	To Mdse. . .	142	13	Sept. 13.	By Mdse.	100	00
Nov. 13.	To Mdse. . .	947	22	Oct. 24.	By Mdse	262	18
Dec. 8.	To Sundries,	1050	00	Nov. 30.	By Mdse.	327	48
				Dec. 21.	By Mdse.	520	75

8. What was due on the following account, July 1, 1850, interest being 6 per cent per year?

Dr. Wm. Barnes in account with James Shedd. Cr.

1850.				1850.			
Jan. 17.	To Sundries,6 mo.	$673	42	Feb. 1.	By Sundries,4 mo.	$237	80
Jan. 28.	To Sundries,4 mo.	542	31	Mar. 5.	By Sundries,3 mo.	492	50
Feb. 13.	To Sundries,6 mo.	237	23	Mar, 21.	By Sundries,6 mo.	873	27
Apr. 22.	To Sundries,3 mo.	720	60	Apr. 25.	By Sundries,3 mo.	594	82
May 10.	To Sundries,2 mo.	54	20	May 27.	By Sundries,4 mo.	376	15
June 23.	To Sundries,3 mo.	133	60	June 2.	By Sundries,2 mo.	142	60
				June 20.	By Sundries,6 mo.	225	00

§ 25. TO FIND THE PRINCIPAL, OR INTEREST, FROM THE AMOUNT, RATE, AND TIME.

When the amount, time, and rate to find the principal or interest are given, we find what part any principal, or (if the interest be required) its interest for the given time, at the given rate, is of its amount, and then take this part of the given amount. The first step towards this is to find the fraction expressing what part interest for the given time, at the given rate, is of its principal. This fraction will *always* be the same part of the given annual rate that the given time is of 1 year, or 360 days; or, *if the rate is* 6 *per cent*, it will equal the fraction expressing the part which the given time is of 200 months, or 6000 days. The amount, of course, will equal the principal, plus the fractional part of it which the interest equals.

Thus interest for 1 yr. 7 mo., or 19 months, at 6 per cent per year $= \frac{19}{200}$ of the principal, and the amount $= \frac{200}{200} + \frac{19}{200} = \frac{219}{200}$ of the principal. Hence the principal $= \frac{200}{219}$ and the interest $\frac{19}{219}$, of the amount for 19 mo. at 6 per cent. (The same fractions would have been obtained by considering the interest to be $\frac{1}{12}$ of $\frac{6}{100}$ of the principal.)

Again: Interest for 19 months at $4\frac{1}{2}$ per cent per year $= \frac{19}{12}$ of $\frac{4\frac{1}{2}}{100} = \frac{19}{12}$ of $\frac{9}{200} = \frac{57}{800}$ of the principal, and the amount =

$\frac{800}{800} + \frac{57}{800} = \frac{857}{800}$ of the principal. Hence the principal $= \frac{800}{857}$, and the interest $\frac{57}{857}$, of the amount for 19 mo. at $4\frac{1}{2}$ per cent.

Again: Interest for 2 yr. 3 mo. 2 da., or 812 days, at 6 per cent per year $= \frac{812}{6000} = \frac{203}{1500}$ of the principal, and the amount $= \frac{1500}{1500} + \frac{203}{1500} = \frac{1703}{1500}$ of the principal. Hence the principal $= \frac{1500}{1703}$, and the interest $= \frac{203}{1703}$, of the amount for 2 yr. 3 mo. 2 da. at 6 per cent. (The same fractions would have been obtained by considering the interest to be $\frac{812}{3600}$ of $\frac{6}{100}$ of the principal.)

Again: Interest for 5 mo. 14 da., or 164 days, at 7 per cent per year $= \frac{164}{360}$ of $\frac{7}{100} = \frac{41}{90}$ of $\frac{7}{100} = \frac{287}{9000}$ of the principal, and the amount $= \frac{9000}{9000} + \frac{287}{9000} = \frac{9287}{9000}$ of the principal. Hence the principal $= \frac{9000}{9287}$, and the interest $= \frac{287}{9287}$, of the amount for 5 mo. 14 da. at 7 per cent.

1. What principal on interest at 6 per cent per year will amount to $884.125 in 1 yr. 2 mo. 10 da.?

Solution. — Since, at 6 per cent per year, interest for 1 day $= \frac{1}{6000}$ of the principal, interest for 1 yr. 2 mo. 10 da., or 430 days, must equal $\frac{430}{6000}$, or $\frac{43}{600}$, of the principal, and the amount must equal $\frac{600}{600} + \frac{43}{600}$, or $\frac{643}{600}$, of the principal. Hence $\frac{1}{600}$ of the principal must equal $\frac{1}{643}$, and the principal itself must equal $\frac{600}{643}$ of the amount. $\frac{600}{643}$ of $884.125 = $825 = principal required. The mere numerical work may be indicated thus: —

1 yr. 2 mo. 10 da. = 430 da. $\frac{430}{6000} = \frac{43}{600}$. $\frac{43}{600} + \frac{600}{600} = \frac{643}{600}$. $\frac{600}{643}$ of $884.125 = $825 = principal required.

2. What principal on interest at 7 per cent per year will amount to $703.551 in 4 mo. 27 da.?

Solution. — At 7 per cent per year, interest for 4 mo. 27 da., or 147 days, $= \frac{147}{360}$ of $\frac{7}{100} = \frac{49}{120}$ of $\frac{7}{100} = \frac{343}{12000}$ of the principal, and the amount $= \frac{12000}{12000} + \frac{343}{12000} = \frac{12343}{12000}$ of the principal. Hence the principal must equal $\frac{12000}{12343}$ of the amount, which in this case is $\frac{12000}{12343}$ of $703.551 = $684. The mere numerical work may be indicated thus: —

4 mo. 27 da. = 147 da. $\frac{147}{360}$ of $\frac{7}{100} = \frac{343}{12000}$. $\frac{343}{12000} + \frac{12000}{12000} = \frac{12343}{12000}$. $\frac{12000}{12343}$ of \$703.551 = \$684 = principal required.

Proof. — The amount of \$684, for 4 mo. 27 da. at 7 per cent equals \$703.551.

3. What principal on interest at 6 per cent will amount to \$569.296 in 8 mo. 16 da.?

4. What principal on interest at 6 per cent will amount to \$573.16 in 2 yr. 9 mo. 10 da.?

5. What principal on interest at 8 per cent will amount to \$922.13 in 1 yr. 4 mo.?

6. What principal on interest at 6 per cent will amount to \$378.82 in 1 yr. 4 mo. 20 da.?

7. What principal on interest at 6 per cent will amount to \$899.944 in 5 mo. 14 da.?

8. What principal on interest at 10 per cent will amount to \$57.72 in 8 mo. 20 da.?

9. What principal on interest at 6 per cent will amount to \$357.68 in 1 yr. 1 mo. 10 da.?

10. What principal on interest at 6 per cent will amount to \$374 in 6 mo. 20 da.?

§ 26. DISCOUNT AND PRESENT WORTH.

Discount, as it is technically called, furnishes the most common application of the processes of the preceding article to the problems of business life. It is obvious that the true present value of a debt due at a future time is that sum of money which, put on interest at the present time, will amount to the given debt at the time it becomes due. Thus, when the rate of interest is 6 per cent per year, a debt of \$106 due in 1 year is worth the same as a debt of \$100 due now; for if the money received on the second debt be put on interest, it will amount to \$106 in 1 year; that is, when the first debt becomes due. A debt due at a future time may be regarded, then, as the amount of

a principal on interest from the present time to the time when the debt will become due. This principal is usually called the *Present Worth* of the debt, and its interest is called the *Discount*, because, if discounted or deducted from the debt, it leaves the present worth.

From this, it follows that to ask what is the present worth of $651, due 6 mo. 20 da. hence, money being worth 6 per cent per year, is equivalent to asking what principal on interest at 6 per cent will amount to $651 in 6 mo. 20 da., and that the solution of the first question is the same as that of the second. To test the correctness of any result, see if the amount of the present worth equals the given debt.

1. What is the present worth of $438.18 due 1 yr. 6 mo. hence, money being worth 6 per cent per year?

Partial Solution. — The present worth required is that sum of money which, put on interest at 6 per cent, will amount to $438.18 in 1 yr. 6 mo., or 18 mo. The interest for 18 mo. $= \frac{18}{200} =$ $\frac{9}{100}$ of the principal, and the amount =, &c., as in last article.

2. What is the present worth of $83.45 due 4 yr. 2 mo. hence, money being worth 6 per cent per year?

3. What is the present worth of $89.88, due 1 year hence, money being worth 7 per cent per year?

4. What is the present worth of $142.56, due 2 years hence, money being worth 4 per cent per year?

5. What is the present worth of $122.94, due 4 mo. 27 da. hence, money being worth 6 per cent per year?

6. What is the present worth of $475.64, due 1 yr. 8 mo. hence, money being worth 6 per cent per year?

7. What is the present worth of $578.50, due 3 yr. 1 mo. 15 da. hence, money being worth 8 per cent per year?

8. What is the present worth of $731.52, due 3 yr. 4 mo. hence, money being worth 6 per cent per year?

9. What is the present worth of $1323.70, due 7 mo. 15 da. hence, money being worth $5\frac{1}{3}$ per cent per year?

10. What is the discount of $195.87, due 1 yr. 5 mo. 19 da. hence, at 6 per cent per year?

Direction. — Find what part of the debt the discount is, and get that part of $195.87. For proof, subtract the discount thus found from $195.87, and see if the interest of the remainder for the given time equals the discount. We may also get the discount by finding the present worth, and subtracting it from the debt.

11. What is the discount of $3946.11, due 2 yr. 5 mo. 15 da. hence, money being worth 6 per cent per year?

12. What is the discount of $6392.43, due 15 mo. 7 da. hence, money being worth 6 per cent per year?

13. What is the discount of $1241.27, due 1 yr. 5 mo. 23 da. hence, money being worth 6 per cent per year?

14. What is the discount of $6255, due 3 yr. 2 mo. hence, money being worth 5 per cent per year?

15. What is the discount of $179.96, due 2 yr. 3 mo. 6 da. hence, at 4 per cent per year?

16. I own a note for $976, payable on demand with interest, and another for $1034.56, payable in just 1 year, with interest afterward. Allowing money to be worth 6 per cent per year, which debt is justly worth the most at the present time, and how much the most? Which will be worth the most at the end of the year? Which will be worth the most at the end of 6 months? Which will be worth the most at the end of 2 years?

17. If two notes are given on the same day, one for a certain sum due at a future time, with interest afterwards, and the other for the true present worth of the first note, payable on demand with interest, their true values will be the same at the time they are given, and also at the time the second becomes due; but at all other times they will differ. At any time between the day of their date and the day when the first note becomes due, the true value of the second note will be greater than that of the first; but at any time after the first note becomes due, the true value of the first note will be greater than that of the second. Show why this is so.

§ 27. MERCHANTS' METHOD OF DISCOUNT.

Business men, instead of allowing the discount obtained by the above method, usually allow a discount equal to the interest of the entire debt from the time of payment to the time when, by its conditions, it would have become due. This, it will be observed, is in accordance with the method adopted at the banks, and is to the advantage of the person advancing the money, as it enables him to pay the debt for a sum less than its *true* present worth. Thus, when money is worth 6 per cent per year, a debt of $106, due in 1 year, is regarded as worth, at the present time, only $106 — .06 of $106 = $106 — $6.36 = $99.64, whereas it is really worth $100.

1. What is the difference between the true present worth of a note for $554.19, due in 3 months without grace, and the present worth of it as obtained by the business method, money being worth 6 per cent per year?

2. I owed a debt of $8692, payable June 1, 1852; but my creditor, offering to allow me discount estimated by the business method at the rate of 6 per cent per year, if I would pay the debt Jan. 1, 1852, I borrowed money for the purpose at 6 per cent interest. June 1, 1852, I paid the amount of the borrowed money. What was my gain by the transaction?

3. Owing a debt of $1545, due in just 6 mo., when money is worth 6 per cent per year, what shall I gain by hiring money enough to pay it now, allowing the usual business discount on the debt, and then paying the borrowed money with interest, at the end of 6 months, when the original debt would otherwise have become due?

4. At 6 per cent per year, what is the difference between the bank discount and the real discount of a note for $2059.40, payable in 60 days? *

* Three days' grace are to be allowed in all such cases.

5. Received for my note of $600, payable in 6 mo., its true present worth. How much more did I receive on it than I should have received at a bank, money being worth 6 per cent? How much interest money shall I have gained, when the note becomes due, over what I should have gained on the present worth, as determined at the bank?

6. For how much must a note, payable in 30 days, be given, that, when discounted at a bank, $900 may be received on it, money being 6 per cent?

Solution. — The money received on a note discounted at a bank equals the sum for which the note is given, minus its interest for the time before it becomes due. Since, at 6 per cent, the interest for 30 days and grace, or 33 days, $= \frac{33}{6000} = \frac{11}{2000}$ of the principal, the sum received must equal $\frac{2000}{2000} - \frac{11}{2000} = \frac{1989}{2000}$ of the face of the note. Therefore the face of the note must be $\frac{2000}{1989}$ of the sum received on it.

$\frac{2000}{1989}$ of $900 $= \frac{\$2000}{1989} \times \frac{100}{900} = \frac{200000}{221} =$ $904.977, or, as it would in practice be considered, $904.98.

The numerical work can be expressed thus: —

$\frac{33}{6000} = \frac{11}{2000}$. $\frac{2000}{2000} - \frac{11}{2000} = \frac{1989}{2000}$. $\frac{2000}{1989}$ of $900 $= \frac{\$2000}{1989} \times \frac{100}{900} =$ $\frac{200000}{221} =$ $904.977; hence face of note $=$ $904.98.

7. How much would be received at a bank on a note for $904.98, payable in 30 days?

NOTE. — The above example suggests the method of proving the 6th.

8. For how much must a note payable in 3 mo. be given, that, when discounted at a bank, $1000 dollars may be received on it, money being worth 6 per cent per year?

* By cancelling 9 from 1989 and 900.

9. For how much must a note payable in 6 mo. be given, that, when discounted at a bank, $1800 may be received on it, money being worth 6 per cent per year?

10. Obtained at a bank, on my note payable in 6 mo., money enough to buy 20 acres of land at $100 per acre. The day my note at the bank became due, I sold the land for $2062.92 cash. Did I gain or lose by the transaction, and how much, money being worth 6 per cent per year?

11. Obtained at a bank, on my note payable in 4 mo., money enough to buy 20 acres of land at $100 per acre. The day the note became due, I sold the land for cash, at such rate that the price of 18 acres was just sufficient to pay the note. How much did I gain by the transaction, money being worth 6 per cent?

12. Obtained at a bank, on my note payable in 3 mo., money enough to buy 20 acres of land at $100 per acre. One month afterwards I sold the land, receiving in payment a note on demand, with interest at 6 per cent per year. I collected the amount of this note the day my note became due at the bank, and found that it took $\frac{9}{10}$ of it to pay the latter. For how much per acre did I sell the land?

13. Obtained at a bank, on my note payable in 3 mo., money enough to buy 20 acres of land at $100 per acre, and at the same time hired of a friend money enough to buy another lot of the same size and price as the first, giving in payment my note payable on demand, with interest at 6 per cent per year. When the note at the bank became due, I sold both lots for cash at the same price per acre, and found that the money received for 16 acres of it was sufficient to pay my note at the bank. How much did I gain by the transactions? How much more on the second lot than on the first?

14. Bought goods to the amount of $864.27 on a credit of 6 mo.; but the seller offering to deduct 5 per cent from the face of the bill if I would pay cash, I hired the requisite amount of money, giving my note payable in 6 mo., with interest at 6 per cent per year, to be reckoned from date. For how much

less than the value of the original bill could I pay the amount of this note?

15. I owed $800, due in 6 mo.; but my creditor offering to deduct 5 per cent of the debt for cash, I paid $380 down. How much did I still owe?

Suggestion. — Since 5 per cent of the debt was to be deducted for cash, the cash payment would be 95 per cent $= \frac{95}{100} = \frac{19}{20}$ of the debt it would cancel; or the debt cancelled would be $\frac{20}{19}$ of the cash paid.

16. I owed $900, payable in 4 mo., but my creditor offering to deduct 4 per cent of the debt for ready money, I paid $696 down. How much did I still owe?

§ 28. TO FIND THE RATE.

Problems in which, the principal, interest, and time being given, we are required to find the rate, rarely occur in business life. The following solutions illustrate the principles which which apply to them.

1. At what rate per cent must $536 be on interest to gain $64.32 in 1 yr. 6 mo.?

Solution. — The principal being equal to 53,600 cents, and the interest to 6432 cents, the interest is $\frac{6432}{53600} = \frac{3}{25}$ of the principal. 1 yr. 6 mo. $= 1\frac{1}{2}$ yr. $= \frac{3}{2}$ yr. If the interest for $\frac{3}{2}$ of a year equals $\frac{3}{25}$ of the principal, the interest for one half of a year must equal $\frac{1}{3}$ of $\frac{3}{25}$, or $\frac{1}{25}$ of the principal, and the interest for 1 yr. must equal twice the last result, or $\frac{2}{25} = \frac{8}{100}$ = 8 per cent of the principal = Ans.

2. At what rate per cent must $648 be on interest to gain $81.873 in 2 yr. 3 mo. 17 da.?

Solution. — The principal being equal to 648,000 cents, and the interest to 81,873 cents, the interest is $\frac{81873}{648000} = \frac{9097}{72000}$ of the principal. 2 yr. 3 mo. 17 da. = 27 mo. 17 da. = 827 da.

If the interest for 827 da. $= \frac{8027}{720000}$ of the principal, the interest for 1 day must equal $\frac{1}{827}$ of $\frac{8027}{720000}$ of the principal, and the interest for 1 yr., or 360 da., must equal 360 times the last result. Expressing the work in proper form, and cancelling, we have

$$\frac{\overset{11}{\cancel{9027}} \times \overset{1}{\cancel{360}}}{\underset{200}{\cancel{720000}} \times \cancel{827}} = \frac{11}{200} = \frac{5\frac{1}{2}}{100}, \text{ or } 5\tfrac{1}{2} \text{ per cent} = \text{Ans.}$$

3. At what rate must $624 be on interest to gain $74.88 in 1 yr. 2 mo. 12 da.?

4. At what rate must $57.25 be on interest to gain $5.038 in 1 yr. 5 mo. 18 da.?

5. At what rate must $855 be on interest to gain $46.55 in 2 yr. 2 mo. 4 da.?

6. At what rate must $64.80 be on interest to gain $6.246 in 11 mo. 17 da.?

§ 29. TO FIND THE PRINCIPAL FROM THE INTEREST.

Problems in which, the interest, rate, and time being given, we are required to find the principal, are, like those in the last article, of rare occurrence.

1. What principal on interest at 6 per cent will gain $37.47 in 1 yr. 3 mo.?

Solution. — At 6 per cent per year, the interest of any principal for 15 mo. $= \frac{15}{200} = \frac{3}{40}$ of the principal. If $37.47 is $\frac{3}{40}$ of the principal, $\frac{1}{40}$ of the principal must equal $\frac{1}{3}$ of $37.47, and the principal must equal 40 times the last result, or $\frac{40}{3}$ of $37.47. Expressing this in proper form for cancelling, we have

$$\frac{\overset{12.49}{\cancel{\$37.47}} \times 40}{\cancel{3}} = \$449.60 = \text{Answer.}$$

2. What principal on interest at 8 per cent will gain $26.18 in 1 yr. 15 da.?

Solution. — Since 1 yr. 4 mo. 15 da. = 16½ mo. = $\frac{16\frac{1}{2}}{12}$ yr. = $\frac{11}{8}$ of 1 yr., the interest must equal $\frac{11}{8}$ of 8 per cent, or 11 per cent of the principal. If $26.18 is 11 per cent of the principal, 1 per cent of the principal must be $\frac{1}{11}$ of $26.18, which is $2.38, and 100 per cent, or the principal, must equal 100 times the last result, which is $238.

3. What principal on interest at 6 per cent will gain $8.73 in 5 mo. at 6 per cent?

4. What principal on interest at 6 per cent will gain $4.77 in 1 yr. 5 mo. 20 da.?

5. What principal on interest at 5 per cent will gain $4.27 in 2 yr. 6 mo.?

6. What principal on interest at 7½ per cent will gain $116.127 in 4 yr. 4 mo. 4 da.?

§ 30. COMPOUND INTEREST.

When interest is to be paid at regular intervals, or, if unpaid, is to be added to the principal, to form a new principal on which interest is to be computed, it is called *Compound Interest.* The following example illustrates this: —

1. What is the compound interest of $784 for 3 yr. 8 mo. at 6 per cent, payable annually?

Solution.

a =	$784.	= principal.
.06 of a = b =	47.04	= interest for 1st year.
a + b = c =	831.04	= amount due at end of 1st year.
.06 of c = d =	49.862	= interest for 2d year.
c + d = e =	880.902	= amount due at end of 2d year.
.06 of e = f =	52.854	= interest for 3d year.
e + f = g =	933.756	= amount due at end of 3d year.
.04 of g = h =	37.350	= interest for 8 mo.
g + h = i =	971.106	= amount due at end of 3 yr. 8 mo.
a =	784.	= principal.
i − a = j =	$187.106	= compound interest for 3 yr. 8 mo.

2. To what sum will $437 amount in 2 yr. 6 mo. at 4 per cent, payable semiannually?

Solution.

	a = $437	= principal.
.02 of a = b =	8.74	= interest 1st 6 mo.
a + b = c =	445.74	= amount at end of 6 mo.
.02 of c = d =	8.915	= interest 2d 6 mo.
c + d = e =	454.655	= amount at end of 12 mo.
.02 of e = f =	9.093	= interest 3d 6 mo.
e + f = g =	463.748	= amount at end of 18 mo.
.02 of g = h =	9.275	= interest 4th 6 mo.
g + h = i =	473.023	= amount at end of 2 yr.
.02 of i = j =	9.460	= interest 5th 6 mo.
i + j = k =	$482.483	= amount due at end of 2 yr. 6 mo.

3. What is the compound interest of $938.63 for 4 yr. 6 mo. at 6 per cent, payable annually?

4. What is the compound interest of $573.32 for 2 yr. 3 mo. at 8 per cent, payable quarterly?

5. What is the compound interest of $1000 for 3 yr. 2 mo. 12 da. at 6 per cent, payable annually?

6. To what sum will $500 amount in 4 yr. 3 mo. at 5 per cent, payable annually?

§ 31. TABLE FOR COMPOUND INTEREST.

USE OF THE TABLE. — The amount of any sum of money, at compound annual interest, for any time and rate mentioned in the table, may be found by multiplying the principal by the appropriate number selected from the table, observing to point off six more places for decimal fractions in the product than there are in the principal. This will be illustrated in the examples and solution following the table: —

Table showing what part of any principal is equal to its amount at 3, 4, 5, 6, 7, and 8 per cent annual compound interest, for any number of years not exceeding 25.

Yrs.	3 per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.	8 per cent.
1.	1.03	1.04	1.05	1.06	1.07	1.08
2.	1.0609	1.0816	1.1025	1.1236	1.1449	1.1664
3.	1.092727	1.124864	1.157625	1.191016	1.225043	1.259712
4.	1.125509	1.169858	1.215506	1.262477	1.310796	1.360489
5.	1.159274	1.216653	1.276281	1.338225	1.402552	1.469328
6.	1.194052	1.265319	1.340096	1.418519	1.500730	1.586874
7.	1.229874	1.315932	1.407100	1.503630	1.605781	1.713824
8.	1.266770	1.368569	1.477455	1.593848	1.718186	1.850930
9.	1.304773	1.423312	1.551328	1.689479	1.838459	1.999005
10.	1.343916	1.480244	1.628894	1.790848	1.967151	2.158925
11.	1.384234	1.539454	1.710339	1.898298	2.104852	2.331639
12.	1.425761	1.601032	1.795856	2.012196	2.252191	2.518170
13.	1.468534	1.665073	1.885649	2.132928	2.409845	2.719624
14.	1.512590	1.731676	1.979931	2.260903	2.578534	2.937194
15.	1.557967	1.800943	2.078928	2.396558	2.759031	3.172169
16.	1.604706	1.872981	2.182874	2.540351	2.952164	3.425943
17.	1.652848	1.947900	2.292018	2.692772	3.158815	3.700018
18.	1.702433	2.025816	2.406619	2.854339	3.379932	3.996019
19.	1.753506	2.106849	2.526950	3.025599	3.616527	4.315701
20.	1.806111	2.191123	2.653298	3.207135	3.869684	4.660957
21.	1.860294	2.278768	2.785962	3.399563	4.140562	5.033834
22.	1.916103	2.369918	2.925260	3.603537	4.430402	5.436540
23.	1.973586	2.464715	3.071524	3.819749	4.740530	5.871464
24.	2.032794	2.563304	3.225100	4.048934	5.072367	6.341181
25.	2.093778	2.665836	3.386355	4.291870	5.427433	6.848475

1. What is the amount of $245.73, for 12 yr. 6 mo. 20 da annual compound interest at 6 per cent.?

Solution. — By the table it appears that the amount of a sum for 12 years, at 6 per cent, compound interest, is 2.012196 times the principal. Multiplying $245.73 by 2.012196, we have $494.4569?308, or omitting the denominations below mills,

$$a = \$494.457 = \text{amount for 12 years.}$$
$$\tfrac{1}{30} \text{ of } a = b = \underline{\quad 16.481} = \text{interest for 6 mo. 20 da.}$$
$$a + b = c = \$510.938 = \text{amount for 12 yr. 6 mo. 20 da.}$$

2. What is the amount of $578.67 for 11 yr. 4 mo. at 3 per cent annual compound interest?

3. What is the amount of $147.43 for 22 yr. 4 mo. 24 da. at 5 per cent annual compound interest?

4. What is the amount of $1467 for 18 yr. at 7 per cent annual compound interest?

5. What is the interest of $1137.38 for 13 yr. 6 mo. at 6 per cent annual compound interest?

6. What is the interest of $328.96 for 9 yr. 3 mo. at 4 per cent annual compound interest?

§ 32. COMMISSION.

Money received for services in buying and selling goods for others is called *Commission*, and is usually reckoned at a certain per cent of the cost of the goods bought, or price of those sold. A merchant who makes it his business to buy and sell on commission is called a *Commission Merchant.*

1. A commission merchant sold goods for a manufacturer to the amount of $568.36, for which he charged a commission of $2\frac{1}{2}$ per cent. What was his commission, and how much will be due to the manufacturer?

Answer. — His commission = .02½ of $568.36 = $14.209, and the sum due the manufacturer = $568.36 — $14.21 = $554.15.

2. A commission merchant bought goods for a country trader to the amount of $738.27, for which he charged a commission of 1½ per cent. What was his commission, and what sum must the trader remit to pay for the goods and commission?

Answer. — His commission = .01½ of $738.27 = 11.07, and the trader must remit $738.27 + $11.07 = $749.34.

3. I sold for Seth Jones 2024 pounds of butter at 19 cents per pound, and 5276 pounds of cheese at 7¼ cents per pound. What was the value of my commission at 2½ per cent on the sales? How much money ought I to pay him?

4. I bought for Francis Jackson 50 pairs of boots at $3.87½ per pair, 100 pairs at $2.75 per pair, 75 pairs at $2.16⅔ per pair, 100 pairs of shoes at $1.56 per pair, and 80 pairs at $1.08 per pair. What was the value of my commission at 2 per cent on the purchase? How much money must Mr. Jackson remit to me to pay for the goods and my commission?

5. I have sent $5000 to my agent in New Orleans, directing him to expend it for cotton, first deducting his commission of 2 per cent on the purchase. What will be his commission, and what will he expend for cotton?

Suggestion. — The $5000 sent includes the sum to be actually invested and my agent's commission of 2 per cent on that sum, and is therefore $\frac{102}{100}$ of the purchase money. Hence the purchase money, or sum to be expended by my agent, is $\frac{100}{102}$ of $5000, and his commission is $\frac{2}{102}$ of $5000. — For proof, see if the sum expended, plus .02 of it, equals $5000.

6. I have received $5600 from my correspondent in St. Louis with directions to expend it in merchandise, first deducting my commission of 2½ per cent on the money expended. How much ought I to expend?

7. My agent in Rochester writes that he has purchased a lot of flour for me at $5 per barrel, and that the entire cost of the flour, including his commission of 2½ per cent, is $1600. How many barrels of flour were purchased?

8. Briggs & Grant sold for Jenks, Clarke, & Co. 1000 brooms at $.25 apiece, for which they charge a commission of 4 per cent. Pursuant to instructions, they invest the balance in sugar at 8 cents per pound, first deducting their commission of 2 per cent on the purchase. How many pounds of sugar did they buy?

9. My agent at Cincinnati writes that he has purchased on my account a lot of provisions, and that his commission of $1\frac{1}{2}$ per cent on the purchase is $13.50. How many dollars worth of provisions has he bought?

10. My agent at New Orleans has purchased for me a lot of cotton at 6 cents per pound, for which he charges a commission of $1\frac{2}{3}$ per cent. His commission amounts to $38.80. How many pounds of cotton has he bought, and how much money must I remit to him to pay for the cotton and his commission?

11. Lewis & Johnson sell for Field & Dean 96 cases of cassimere, each case containing 276 yards, at $1.25 per yard, for which they charge a commission of $3\frac{1}{2}$ per cent. They receive instructions from Field & Dean to remit them half of the net proceeds, and to invest the remainder, after deducting a commission of $1\frac{1}{4}$ per cent on the purchase, in wool, at 50 cents per pound. What was the value of the cash remitted to Field & Dean? How many pounds of wool were bought?

12. Haskell & Latham sell for me, at auction, goods to the amount of $8732. Their charges are as follows: Auction tax, 1 per cent; commission, 4 per cent; guaranteeing the sales, 3 per cent; advertising, $23.25; truckage and storage, $11.25. How much money will be due me?

Suggestion. — The auction tax, commission, and guaranty amount to 8 per cent of the value of the goods, which, together with the other expenses, must be deducted from the value of the goods, to leave the sum due me.

13. Miles & Holt have sold a lot of goods for me at auction, for which their charges are 1 per cent for auction tax; $3\frac{1}{2}$ per cent for commission; 4 per cent for guaranteeing the sales;

$37.29 for advertising and other expenses; and they find that, after deducting these charges, there is due me a balance of $3977.73. What was the value of the goods sold? Ans. $4388.

14. My agent in New Orleans has sold for me a lot of goods for $3375, on a credit of 6 months, and got the note discounted at a bank.* If he charges 4 per cent for services in selling, and 3 per cent for guaranteeing the note at the bank, how much ought he to remit to me?

Suggestion. — Since the note was given for what the goods sold for, his commission and guaranty are together 7 per cent of the price of the goods, or face of the note; therefore he ought to remit to me the sum received from the bank, minus 7 per cent of the sum received for the goods.

§ 33. STOCKS.

Money invested in any property designed to yield an income is called *Stock*. The money invested by a man in his business is called his *Stock in Trade;* that invested in government securities, bonds, &c., is called *Government Stock*.

The *Capital Stock* of any incorporated company is the money paid in by its members for the general purposes for which the company was formed. It is divided into equal parts, called *Shares*. Any person owning one or more of these shares is a *Stockholder*, or member of the corporation. *Stocks* is a general term applied to the shares themselves, which may be bought and sold like any other property.

The value at which the shares of any corporation are rated in estimating its capital stock, that is, their first or original value, is called their *Nominal Value*, or their *Par Value*, and is always the same. The price which they will bring, if exposed for sale, is their *True* or *Real Value*, and is different at different times. If the real value equals the par value, the stocks are at par; if

* It will be remembered that bank interest in Louisiana is 6 per cent per year.

it be greater, they are above par, and sell at a *Premium*, or *Advance;* if it be less, they are below par, and sell at a *Discount.*

The profits accruing to the corporation, if any, are at intervals distributed among the members in proportion to the number of shares each holds, and are then called *Dividends.* The dividends are usually reckoned as a certain per cent of the par value of the shares.

1. How much will 11 shares Fall River Railroad stock cost, at an advance of 6 per cent, the par value being $100 per share?

Solution.

$1100 = par value of 11 shares.
66 = 6 per cent premium.
$1166 = real value, or required cost.

2. How much will 11 shares of Providence Railroad stock cost, at a discount of 6 per cent, the par value being $100 per share?

Solution.

$1100 = par value of 11 shares.
66 = 6 per cent discount.
$1034 = real value, or required cost.

3. How much will 53 shares Suffolk Bank stock cost at an advance of 23 per cent, the par value being $100 per share?

4. How much will 43 shares Vermont Central Railroad stock cost, at a discount of 78 per cent, the par value being $50 per share?

5. How many shares of stock at an advance of 5 per cent on the par value of $100 per share, can be bought for $1995?

6. How many shares of stock at a discount of 5 per cent from the par value of $100 per share, can be bought for $1805?

7. A broker paid $1776.50 for stock at an advance of 4½ per cent. What was the par value of the stock bought?

8. A broker paid $4850 for bank stock, at a discount of 3 per cent. What was the nominal value of the stock bought?

9. A broker sold on consignment 376 shares bank stock, par

value $100 per share, at an advance of 7 per cent. He charged 25 cents per share for his services. How much ought he to remit to the person consigning the stock?

10. I bought 40 shares of railroad stock, at 33 per cent below par, and after keeping them 10 months, sold them at 20 per cent below. How much did I gain on them, allowing that I hired money for the investment at 6 per cent interest, and that the par value of the shares was $100 each?

11. Alfred E. Potter bought 10 shares of bank stock at par, which was $50 dollars per share. At the end of 3 months he received a dividend of 4 per cent, and at the end of 9 months he received another of 3 per cent. At the end of 1 yr. he sold the stock at an advance of 3 per cent. Money being worth 6 per cent per year, how much did he gain by the transactions?

NOTE. — Interest should be reckoned on the dividends from the time they were made.

12. Moses E. Fuller bought 18 shares of bank stock at an advance of 8 per cent on their par value of $100 per share. Six months afterwards, and at the end of every subsequent 6 months, he received a dividend of 4½ per cent. At the end of 2 yr. 3 mo. he sold the stock at a premium of 12 per cent. Money being worth 6 per cent per year, compound interest, how much did he gain by the speculation?

13. Crocker & Guild sent $972.63 to a stock broker, directing him to invest it in Fall River Railroad stock. He bought the stock at a premium of 7 per cent, the par value being $100 per share, and he charged a commission of 1 per cent on the money invested. How many shares did he buy?

14. I paid $7398 for stock at 10 per cent below par, and some time after sold the stock at 10 per cent above par. How much did I gain on it?

15. Mr. Hamblin bought stock at 10 per cent above par, but he was obliged to sell it at 10 per cent below par. Allowing that he lost $138.40 on it, what did he pay for it?

§ 34. PROFIT AND LOSS.

Most problems in profit and loss come under one of three classes, viz.: —

1. Those in which it is required to find for what price articles must be sold that their owner may gain or lose a certain per cent of their cost.

2. Those in which it is required to find the cost, or some per cent of the cost, of articles on which a certain per cent will be gained or lost by selling them at a given price.

3. Those in which it is required to find what per cent of the given cost will be gained or lost by selling articles at a given price.

They involve similar principles to those involved in other examples in percentage and fractions.

1. A speculator bought a lot of land for $2473. For how much must he sell it to gain 25 per cent of its cost?

Suggestion. — He must sell it for the cost, plus 25 per cent of the cost; or, since 25 per cent $= \frac{1}{4}$, he must sell it for the cost, plus $\frac{1}{4}$ of the cost.*

2. Mr. Huntington bought a lot of grain at 54 cents per bushel; but it being damaged, he was obliged to sell it at a loss of $16\frac{2}{3}$ per cent. For how much per bushel did he sell it?

Suggestion. — He must have sold it for the cost, minus $16\frac{2}{3}$ per cent of the cost; or, since $16\frac{2}{3}$ per cent $= \frac{1}{6}$, he must have sold it for the cost, minus $\frac{1}{6}$ of the cost.

3. Mr. Shelley bought a lot of sugar at 8 cents per pound. For how much per pound must he sell it to gain $12\frac{1}{2}$ per cent?

4. For how much per yard must cloth, costing $2.50 per yard, be sold, to gain 10 per cent on its cost?

---

* When the given per cent can be reduced to a convenient vulgar fraction, as in this example, it is ordinarily best to use the vulgar fraction instead of the given per cent.

5. Mr. Anthony bought raisins at $6.25 per cask, and sold them at a loss of 5 per cent. For how much per cask did he sell them?

6. A trader bought a cask of oil, containing 83 gallons, for $103.73. For how much per gallon must he sell it, to gain 25 per cent on its cost?

7. I bought cloth at $2 per yard, and sold it at $2.33 per yard. What per cent of its cost did I gain?

Solution. — Since I gave $2 per yard for the cloth, and received $2.33, I received $.33 per yard more than I gave; and as $.33 = $\frac{33}{200}$ of $2, my gain must be $\frac{33}{200} = 16\frac{1}{2}$ per cent of the cost.

8. What per cent shall I lose by selling molasses, which cost me $.30 per gallon, for $.25 per gallon?

Solution. — Since I bought the molasses for 30 cents, and sold it for 25 cents, per gallon, I lost 5 cents per gallon on it; and, as 5 cents = $\frac{5}{30} = \frac{1}{6}$ of 30 cents, I lost $\frac{1}{6}$, or $16\frac{2}{3}$ per cent, of the cost.

9. How much per cent shall I gain by selling flour, which cost me $6.50 per barrel, for $7.215 per barrel?

10. Mr. Winsor bought flour at $7.50, and sold it at $6.75 per barrel. How much per cent did he lose?

11. What per cent shall I gain by selling linen cloth, which cost 45 cents per yard, at 50 cents per yard?

12. What per cent shall I lose by selling linen cloth, costing 50 cents per yard, at 45 cents per yard?

13. Mr. Stowe bought cloth at $.75 per yard, and sold it at $.80 per yard. What per cent did he gain?

14. Mr. Miles bought cloth at $.80 per yard, and sold it at $.75 per yard. What per cent did he lose?

15. A merchant bought some molasses at $.20 per gallon, and some oil at $1.16 per gallon. He sold the molasses at $.25 per gallon, and the oil at such rate that he gained the same per cent on it that he gained on the molasses. For how much per gallon did he sell the oil?

16. I gained $17.28 by selling a lot of sugar for 16 per cent more than it cost. How many dollars did it cost, and for how many did I sell it?

Suggestion. — $17.28 = 16 per cent, or $\frac{4}{25}$ of its cost, = $\frac{16}{116}$ = $\frac{4}{29}$ of the price for which I sold the sugar.

17. Mr. Kent bought a lot of apples, and sold them for 20 per cent more than they cost, by which he gained $24.80. How much did they cost him, and for how much did he sell them?

18. Mr. Kilburn sold 43 barrels of apples for $6.45 less than they cost him, thereby losing 10 per cent of their cost. What did they cost him, and what did he get per barrel for them?

19. Mr. Thurbur sold 146 yards of cloth for $71.54 more than it cost him, thereby gaining 14 per cent. How much did he receive per yard for it?

20. Logee & Drown sold a large lot of goods for $6700.43¼, thereby gaining 18 per cent on their cost. How much did the goods cost them?

Suggestion. — Since they gained 18 per cent of the cost, $6700.43¼ must be 118 per cent, or $\frac{118}{100}$ of the cost.

21. What is the cost of a lot of goods on which 15 per cent will be gained by selling them for $288.65?

22. What is the cost of a lot of goods on which 8 per cent will be gained by selling them for $622.215?

23. What is the cost of a lot of goods on which 7 per cent will be lost by selling them for $442.68?

Suggestion. — Since 7 per cent will be lost by the sale, $442.68 must equal the cost, minus 7 per cent of the cost, which is 93 per cent of the cost.

24. What is the cost of a lot of goods on which 30 per cent will be lost by selling them for $874.846?

25. What is the cost of a lot of goods on which 9 per cent will be lost by selling them for $9009?

26. I sold a lot of goods for \$72.625, which was 17 per cent less than cost. What was their cost?

27. A man bought corn at 50 cents per bushel, for which he asked 25 per cent more than it cost him; but it falling in price, he was obliged to sell it for 25 per cent less than his asking price. Did he gain or lose, and how much per cent? How much on each bushel?

Solution. — Since his asking price was 125 per cent, or $\frac{5}{4}$ of the cost, and his selling price was 75 per cent, or $\frac{3}{4}$ of his asking price, his selling price must have been $\frac{3}{4}$ of $\frac{5}{4}$, or $\frac{15}{16}$ of the cost. Therefore he lost $\frac{1}{16}$, or $6\frac{1}{4}$ per cent, of the cost; and since each bushel cost \$.50, his loss per bushel must have been $\frac{1}{16}$ of \$.50, which is \$.03$\frac{1}{8}$.

28. A merchant asked for a lot of goods $12\frac{1}{2}$ per cent more than they cost him, but was obliged to deduct $12\frac{1}{2}$ per cent from his asking price. What part of the cost did he lose?

29. I asked for a lot of cloth 10 per cent more than it cost me, but was obliged to deduct 10 per cent from my asking price. What per cent of its cost did I lose?

30. A watch dealer bought a watch for \$75, and asked for it $33\frac{1}{3}$ per cent more than it cost. He was obliged to sell it for 10 per cent less than his asking price. What per cent did he gain on the investment? How many dollars?

Ans. He gained 20 per cent of the cost, which was \$15.

31. I asked for a quantity of grain 20 per cent more than it cost me, but was obliged, in selling it, to deduct 10 per cent from my asking price. What per cent of the cost did I gain?

32. I bought a lot of goods for \$6478, for which I asked 25 per cent more than they cost; I was, however, obliged to sell them for 15 per cent less than my asking price. How many dollars did I gain on them?

33. A merchant asked for a quantity of goods 25 per cent more than they cost him, but was obliged to sell them for $12\frac{1}{2}$ per cent less than his asking price. He gained \$98.70 by the

transaction. How much did the goods cost? For how much did he sell them? What was his asking price?

Partial Solution. — Since his asking price was 125 per cent, or $\frac{5}{4}$ of the cost, and his selling price was $87\frac{1}{2}$ per cent, or $\frac{7}{8}$ of his asking price, he must have sold the goods for $\frac{7}{8}$ of $\frac{5}{4}$, or $\frac{35}{32}$ of their cost, which shows a gain of $\frac{3}{32}$ of the cost. Therefore $98.70 = $\frac{3}{32}$ of the cost, and $\frac{1}{32}$ of the cost = $\frac{1}{3}$ of $98.70, and the cost itself = $\frac{32}{3}$ of $98.70. He sold the goods for $\frac{35}{3}$ of $98.70, and the asking price = $\frac{5}{4}$ of the cost = $\frac{5}{4}$ of $\frac{32}{3}$ = $\frac{40}{3}$ of $98.70.

34. A merchant asked for a lot of cloth 20 per cent more than it cost him; but being in want of money, he sold the cloth for 25 per cent less than his asking price. Allowing that he lost $.275 per yard, for how much per yard did he sell it?

35. I sold a lot of goods for $4268, which was $8\frac{1}{3}$ per cent less than my asking price. My asking price was 44 per cent more than the cost of the goods. How much did they cost?

36. By selling flour at $6.65 per barrel, I shall lose 5 per cent of its cost. For how much per barrel must I sell it to gain 5 per cent?

Suggestion. — $6.65 = 95 per cent of the cost. Hence 105 per cent of the cost must equal $\frac{105}{95}$ of $6.65.

37. By selling land at $88.14 per acre I shall gain 13 per cent. For how much must I sell it to gain 20 per cent?

38. By selling a lot of goods for $113.75 I lost 9 per cent. For how much ought I to have sold it to gain 9 per cent?

39. I bought 400 bushels of grain, and sold half of it at 75 cents per bushel, which was 20 per cent more than it cost. I sold the remainder at an advance of 25 per cent on the cost. For how much per bushel did I sell the last lot? How many dollars did I gain?

40. What must be the asking price of cloth costing $2.55 per yard, that I may fall 10 per cent on it, and still gain 14 per cent on the cost?

Suggestion. — Since the selling price is to be 90 per cent, or $\frac{90}{100}$ of the asking price, and 114 per cent, or $\frac{114}{100}$ of the cost, 1 per cent of the asking price will be $\frac{1}{90}$ of $\frac{114}{100}$ of the cost, and the asking price itself will be 100 times the last result, or $\frac{100}{90}$ of $\frac{114}{100}$ of the cost $= \frac{114}{90}$ of the cost.

41. What must be the asking price of cloth costing $3.29 per yard, that I may deduct $12\frac{1}{2}$ per cent from it, and still gain $12\frac{1}{2}$ per cent on the cost?

42. What must be the asking price of boots costing $2.75 per pair, that I may fall $16\frac{2}{3}$ per cent on it, and still gain 20 per cent on their cost?

43. What must be the asking price of shawls costing $7.56 apiece, that I may fall 10 per cent on it, and still gain 15 per cent on their cost?

44. If by selling cloth at $2.34 per yard, I gain 4 per cent of its cost, what per cent shall I gain by selling it at $2.79 per yard?

Solution. — Since $2.34 = 4 per cent more than the cost, it must equal 104 per cent of the cost. If $2.34, or 234 cents = 104 per cent of the cost, 1 cent must equal $\frac{1}{234}$ of 104 per cent, and $2.79, or 279 cents, must equal 279 times the last result. This, expressed in form for cancelling, is

$$\frac{279}{234} \text{ of } 104 \text{ per cent} = \frac{\overset{4}{\cancel{104}} \times \overset{31}{\cancel{279}}}{\underset{26}{\cancel{234}}} = 124 \text{ per cent of the cost, or}$$

24 per cent more than the cost. Therefore he would gain 24 per cent by selling the cloth at $2.79 per yard.

45. If, by selling cloth at $4.37 per yard, I gain 15 per cent of its cost, what per cent shall I gain by selling it at $4.94 per yard?

46. If, by selling flour at $6.75 per barrel, I gain 8 per cent of its cost, what per cent shall I lose by selling it at $6 per barrel?

47. If, by selling oil at $1.254 per gallon, I lose 5 per cent

of its cost, what per cent shall I gain by selling it at $1.584 per gallon?

48. If, by selling a lot of land for $5728.93, I lose 11 per cent of its cost, how many dollars shall I gain by selling it for $7724.40?

49. If, by selling my house for $9843.90, I shall gain 14 per cent of its cost, how many dollars shall I gain by selling it for $10534.70?

50. I bought 96 acres of land at $84 per acre, and sold $\frac{3}{4}$ of it for the cost of the whole. What per cent did I gain on the part sold?

Solution. — Since the part sold was $\frac{3}{4}$ of the land bought, it must have cost $\frac{3}{4}$ as much as was received for it; hence its selling price must have been $\frac{4}{3}$ of its cost, and there must have been a gain of $\frac{1}{3}$, or $33\frac{1}{3}$ per cent, of its cost.

51. I bought 28 tons of iron at $48 per ton, and sold $\frac{5}{7}$ of it for the cost of the whole. What per cent did I gain on the part sold?

52. I bought 83 barrels of beef at $12.50 per barrel, and was obliged to sell it for what $\frac{4}{5}$ of it cost. What per cent did I lose?

53. I bought 25,000 feet of boards at $2.25 per thousand, and sold $\frac{1}{2}$ of them for what $\frac{2}{3}$ of them cost. What per cent did I gain on the part sold?

54. I bought 63 kegs of nails, each keg containing 100 pounds, at $4\frac{1}{2}$ cents per pound, and sold $\frac{2}{3}$ of them for what $\frac{1}{2}$ of them cost. What per cent did I lose on the part sold?

55. I sold $\frac{1}{2}$ of a lot of land for the cost of $\frac{3}{4}$ of the lot, and the remainder for $\frac{1}{2}$ of what I sold the first part for. What per cent of its cost did I gain on the entire lot?

56. I sold $\frac{3}{5}$ of a lot of land for 20 per cent more than it cost, and the remainder for 20 per cent less than it cost. What per cent did I gain on the whole?

57. I sold $\frac{3}{4}$ of a cask of wine for $36, which was 25 per cent more than the part sold cost. I then sold the remainder at an advance of 20 per cent on its cost. What per cent of the cost of the cask did I gain?

58. I bought a lot of groceries for \$1728. I sold $\frac{1}{3}$ of them at an advance of 30 per cent, $\frac{1}{6}$ of them at an advance of 12 per cent, $\frac{1}{4}$ of them at a loss of 24 per cent, and the remainder at cost. Did I gain or lose, and what per cent of the cost of the whole?

Ans. I gained 6 per cent.

59. I bought a lot of groceries, and after selling $\frac{1}{5}$ of them at an advance of 15 per cent, $\frac{1}{4}$ of them at a loss of 8 per cent, $\frac{1}{2}$ of them at an advance of 18 per cent, and the rest at cost, I found that I had received \$132.70 more than the cost of the entire lot. How many dollars did it cost?

60. A. & M. J. Miles bought of Cragin & Cleveland, for cash, goods to the amount of \$423.75, and the same day sold them at an advance of 16 per cent, receiving in payment a note on 3 months. This note they got discounted at a bank at the rate of 6 per cent per year. How much did they gain on the goods?

61. Armington, Horswell, & Kilburn bought of Godding, Briggs, & Co. goods to the amount of \$1000, payable in 6 months, without grace. One month afterwards they sold the goods for cash, at an advance of 10 per cent, and immediately put the money at interest at 6 per cent. When the 6 months had expired, they collected the amount of the money they had lent, and paid the bill due Godding, Briggs, & Co. Did they gain or lose, and how many dollars?

62. Eddy & Barrows bought of Haven, Lewis, & Dean a cask of oil, containing 132 gallons, at \$1.25 per gallon, and gave in payment their note payable in 3 months. They sold the oil for cash, at 8 per cent advance. How many dollars did they gain by the transactions, money being worth 6 per cent per year, and their note being worth the sum which would be received on it if discounted at a bank?

63. A speculator borrowed \$2000 at 9 per cent per year, with which he purchased a lot of cotton. He sold the cotton at an advance of 25 per cent on the cost. With the money

received for the cotton, he purchased railroad stock. At the end of 3 mo. 17 da. from the time he borrowed the $2000, he sold his railroad stock at a loss of 5 per cent, and immediately paid the amount of his borrowed money. Did he gain or lose by the transactions, and how much?

64. An importer sold cloth to a wholesale dealer, and gained 10 per cent of what it cost him. The wholesale dealer sold it to a retail dealer at an advance of 10 per cent on what it cost him. The retail dealer sold it at an advance of 20 per cent on what it cost him. Now, allowing that the retail dealer received $726 for the cloth, how much did it cost the importer?

65. I bought a lot of coffee at 12 cents per pound. Allowing that 5 per cent of the coffee will waste in weighing it out, and that 10 per cent of the sales will be bad debts, for how much per pound must I sell it to make a clear gain of 14 per cent on the cost?

Ans. 16 cents per pound.

66. What must be the asking price of raisins costing $7.29 per cask, that I may fall 10 per cent from it and still gain 10 per cent on the cost, allowing that 10 per cent of the sales will be bad debts?

67. June 1, 1852, I bought for cash 500 casks of oil, each cask containing 42 gallons, at $1.10 per gallon. Oct. 1, 1852, I sold it on 3 months' credit, at a price per gallon equal to 125 per cent of its cost per gallon, deducting 5 per cent of the whole quantity of oil for leakage. I immediately got the note received for the oil discounted at a bank. Allowing that I paid 10 cents per cask for truckage, and $25 for storage and other expenses, and that money was worth 6 per cent per year, did I gain or lose, and how many dollars?

68. I bought 8 casks of oil, each containing 133 gallons, at $1.20 per gallon, and paid $5.32 for having it brought to my store. Allowing that there will be a waste of 5 per cent in measuring, that 3 per cent of my sales will be bad debts, and that it will cost 1 per cent of the remainder to collect it, for

how much per gallon must I sell it to make a net gain of 33 per cent on its cost at my store, nothing being allowed for interest?

69. I sent to my agent in Boston a lot of flour, which he sold for $6075, charging a commission of 2 per cent on the sales. He invested the remainder, after deducting his commission of 1¼ per cent on the purchase, in cloths, which he shipped to my agent in Savannah. The latter sold them at an advance of 25 per cent on the cost, charging a commission of 5 per cent on the sales, and invested the balance, after deducting a commission of 2 per cent on the purchase, in cotton. The cotton was shipped to my agent in Boston, who sold it at an advance of 20 per cent on its cost, charging a commission of 1¾ per cent. Allowing that the expenses of freight, insurance, &c., were $1000, what was my gain?

70. Jan. 1, 1851, my agent in Buffalo bought for me 1000 barrels of flour at $4.00 per barrel, for which he charged a commission of 1 per cent. On the 3d of January, I sent him cash to pay for the flour and his commission. It cost me $1 per barrel to have the flour transported to Boston, and I incurred other expenses upon it to the amount of $20. Feb. 1, 1851, I sold the flour to J. Smith & Co., at an advance of 25 per cent on its entire cost, receiving in payment half cash, and their note payable in 6 months for the remainder. I had their note discounted at a bank; but before it became due they failed, so that when it became due, I, as indorser, was obliged to pay it. Jan. 1, 1852, I settled with J. Smith & Co., receiving 50 cents on each dollar they owed me. Allowing that money was worth 6 per cent per year, and that I paid the freight and other expenses of the flour on the 15th of January, what was the amount of my loss?

§ 35. PARTNERSHIP.

Two or more persons, uniting for the purpose of carrying on business together, form what is called a *Partnership*, *Firm*, or *Company*. The capital invested by them is called their *Stock in Trade*. It is evident that the profit or loss made by the com-

pany should be shared among its members in proportion to what the use, or interest, of each man's stock for the time it was invested is worth.

When the stocks of the several partners are invested for the same length of time, their use, or interest, will be proportioned to the stocks themselves, and hence each partner's gain or loss will be the same part of his stock that the entire gain or loss is of the entire stock; or it will be the same part of the entire gain or loss that his stock is of the entire stock. The following example and solution will illustrate this: —

1. A, B, & C trade in company. A puts in $250, B puts in $750, and C puts in $500. At the end of 6 months they find that they have gained $472.50. What is each man's share of the gain?

1st Solution. — Since A's stock = $250, B's = $750, and C's = $500, the entire stock = $250 + $750 + $500 = $1500; and as the gain = $472.50, it must equal $\frac{472.50}{1500.00}$, $\frac{63}{200}$ of the stock. Therefore each man's gain will be $\frac{63}{200}$ of his stock, which gives for A's gain $78.75, for B's gain $236.25, and for C's gain $157.50.

2d Solution. — Since A's stock = $250, B's = $750, and C's = $500, the entire stock must equal $1500, of which A's stock = $\frac{250}{1500} = \frac{1}{6}$, B's = $\frac{750}{1500} = \frac{1}{2}$, C's = $\frac{500}{1500} = \frac{1}{3}$. Therefore A should have $\frac{1}{6}$, B should have $\frac{1}{2}$, and C should have $\frac{1}{3}$ of the gain. $\frac{1}{6}$ of $472.50 = $78.75 = A's share; $\frac{1}{2}$ of $472.50 = $236.25 = B's share; $\frac{1}{3}$ of $472.50 = $157.50 = C's share.

2. X, Y, & Z traded in company for 1 year. X put in $1000, Y put in $1500, and Z put in $2000. At the end of the year they found that they had gained $1800. What was each man's share of the gain?

3. Henry Read, Alfred Briggs, and Samuel Eddy formed a partnership under the name of Henry Read & Co. Read put in $800, Briggs put in $1050, and Eddy put in $1250. They lost $800. What was each one's share of the loss?

4. G. W. Henshaw, G. T. Rodman, D. C. Smith, and S. Olney formed a partnership under the title of Henshaw, Rodman, & Co. Henshaw put into the firm $650, Rodman $720, Smith $840, and Olney $500. They gained $930. What was each man's share of the gain?

5. George Huntington, Joseph Kent, James Woods, and Alfred Holt bought a ship for $40,000, of which Huntington paid $18,000, Kent $4500, Woods $6000, and Holt the remainder. In her first voyage she cleared $25,000. What was each man's share of the gain?

6. A man, failing in business, finds that he owes A $424, B $638, C $197, D $338, and E $574, and that his whole available property amounts only to $1173. How much ought he to pay to each creditor?

Suggestion. — This question is to be performed on the same principle as were the preceding ones. Since he owes $2171, and has but $1173, he can pay but $\frac{1173}{2171}$ of his debts. Therefore he ought to pay A $\frac{1173}{2171}$ of $424, B $\frac{1173}{2171}$ of $638, &c.

7. The stock of a bankrupt is valued at $1200, and he owes $4200. How many dollars ought he to pay the person to whom he owes $546? to whom he owes $338.73?

8. A, B, C, and D agree to cut 500 cords of wood for $300. When the job is finished, they find that A has cut 125 cords, B 100 cords, C 150 cords, and D the rest. How many dollars ought each to receive?

9. A and B traded in company. A put in $200, and B put in $300. A's share of the gain was $84.56. What was B's share?

10. A and B traded in company, and gained $348, of which B's share was $261. If A's stock was $175, what was B's stock, and A's share of the gain?

11. Samuel Greene and Joseph Irons traded in company. Greene paid in 3 times as much of the stock as Irons, and they gained $1176. What was each one's share of the gain?

Suggestion. — Since Greene paid in 3 times as much as Irons, both together must have paid in 4 times as much as Irons. Therefore Irons paid in $\frac{1}{4}$, and Greene $\frac{3}{4}$ of the stock.

12. William Balch and Joseph Adams bought a ship together, Balch paying in twice as much money as Adams. At the end of 1 year they sold her, and found that they had realized a profit of $15,000 from her. What was each partner's share?

13. Anderson and Parker, after trading in company for 2 years, found that their profits have been $2400. Allowing that Anderson's stock was $\frac{2}{3}$ of Parker's, how many dollars of the profit ought each to have?

14. A, B, and C traded in company. A put in $\frac{1}{4}$ of the stock, B put in $\frac{1}{2}$ of it, and C put in the rest. On dividing the gain, they found that C's share of it was $321. What was the gain of each of the other partners?

15. William Hall, Edward Johnson, and Henry Whiting traded in company, and gained $6534, of which Johnson's share was $1089. If Johnson's and Whiting's stock was together equal to twice Hall's, what was Hall's share of the gain? What was Johnson's share?

16. A, B, and C trade in company, and gain $100, of which A has $12.50, B has $25, and C has $62.50. C put in $21 more of the original stock than A and B together. What was the original stock?

17. Reuben Aldrich and George Guild bought cloth together, Aldrich paying $6 more than $\frac{1}{2}$ of its cost. They sold the cloth at such rate that Aldrich's share was $39, and Guild's share was $36. What did the cloth cost?

Suggestion. — Since Aldrich had $\frac{39}{75}$ of the gain, he must have paid $\frac{39}{75}$ of the cost of the cloth, which, by the conditions of the question, was $6 more than half of the cost.

18. A and B bought cloth together, A paying $4 more than $\frac{3}{5}$ of its cost. They sold the cloth at such rate that A's share of the gain was $101, and B's share was $49. What did the cloth cost?

19. Lyman Richards and John Dexter traded in company, Richards paying in $9 less than $\frac{3}{5}$ of the whole stock. They gained $200, of which Richards's share was $117. What was the original stock of each?

20. Bernard Farwell and Francis Dana traded in company. Farwell's stock was $860, and Dana's was $420. On dividing their profits, they found that Farwell's share was $4 more than twice Dana's. How many dollars did each gain?

21. A small estate belonged to a large number of heirs: 2 members of the family of A each owned $\frac{1}{150}$ of the estate; 4, of the family of B, each owned $\frac{1}{300}$ of it; 4, of the family of C, each owned $\frac{1}{75}$ of it; 2, of the family of D, each owned $\frac{1}{30}$ of it; 4, of the family of E, each owned $\frac{2}{75}$ of it; 3, of the family of F, each owned $\frac{2}{525}$ of it; 4, of the family of G, each owned $\frac{2}{375}$ of it; 6, of the family of H, each owned $\frac{2}{105}$ of it; 3, of the family of I, each owned $\frac{2}{45}$ of it. Mr. Byram, as agent for the above-named individuals, sold their interest in the estate for $350. How many dollars ought he to give to each?

Answer.

$ 3.123 to each member of A's family.
$ 1.562 to each member of B's family.
$31.234 to each member of C's family.
$15.617 to each member of D's family.
$12.493 to each member of E's family.
$ 1.785 to each member of F's family.
$ 2.499 to each member of G's family.
$ 8.924 to each member of H's family.
$20.823 to each member of I's family.

NOTE. — The above example is a statement of transactions which actually occurred. It was brought to the author for solution, by the agent of the parties.

§ 36. PARTNERSHIP ON TIME.

In dividing the gain or loss among the partners, when their shares of the stock are invested for unequal times, it becomes necessary to consider both the stock and the time, or to consider the interest of each man's stock for the time it was in trade. The following examples and solutions will illustrate this: —

1. A, B, and C traded in company. A put in \$750 for 10 mo., B put in \$375 for 12 months, and C put in \$1125 for 16 months. They gained \$860. What was each man's share of the gain?

1st Solution.

Interest of \$ 750 for 10 mo. = \$37.50 = interest of A's stock.
Interest of \$ 375 for 12 mo. = \$22.50 = interest of B's stock.
Interest of \$1125 for 16 mo. = \$90.00 = interest of C's stock.
\$150.00 = interest of whole.

Therefore A should have $\frac{3750}{15000}$, or $\frac{1}{4}$, of the gain, = \$215.
B should have $\frac{2250}{15000}$, or $\frac{3}{20}$, of the gain, = \$129.
C should have $\frac{9000}{15000}$, or $\frac{3}{5}$, of the gain, = \$516.

2d Solution.

The use of \$ 750 for 10 mo. is worth the use of \$ 7500 for 1 mo.
The use of \$ 375 for 12 mo. is worth the use of \$ 4500 for 1 mo.
The use of \$1125 for 16 mo. is worth the use of \$18000 for 1 mo.
Use of whole stock is worth the use of \$30000 for 1 mo.

Therefore A should have $\frac{7500}{300000}$, or $\frac{1}{4}$, of the gain, = \$215.
B should have $\frac{4500}{300000}$, or $\frac{3}{20}$, of the gain, = \$129.
C should have $\frac{18000}{300000}$, or $\frac{3}{5}$, of the gain, = \$516.

When the stocks of the several partners are convenient fractional parts or multiples of each other, a very neat solution can be given. Thus, in the above example, by noticing that B's stock equals $\frac{1}{2}$ of A's, and that C's stock equals $\frac{3}{2}$ of A's, we may have the following: —

3d Solution. — The use of A's stock 10 mo. = use of 10 times A's stock for 1 mo. = A's.

B's, or $\frac{1}{2}$ of A's stock, 12 mo. = use of $\frac{1}{2}$ 12, or 6 times A's stock for 1 mo.

C's, or $\frac{3}{2}$ of A's stock, 16 mo. = use of $\frac{3}{2}$ 16, or 24 times A's stock for 1 mo.

Use of whole = use of 40 times A's stock for 1 mo.

Therefore A should have $\frac{10}{40}$, or $\frac{1}{4}$; B $\frac{6}{40}$, or $\frac{3}{20}$; and C $\frac{24}{40}$ or $\frac{3}{5}$, of the gain, which will give the same answer as before.

2. A, B, and C traded together. A put in $600 for 17 mo., B put in $800 for 18 mo., and C put in $800 for 20 mo. They gained $864. What was each man's share?

3. Ritchie, Edwards, and Boyden traded together, and gained $900. If Ritchie put in $800 for 7 mo., Edwards put in $900 for 8 mo., and Boyden put in $700 for 9 mo., what part of the gain ought each to receive?

4. Charles French, Francis Baker, and Otis Atherton traded in company under the name of Charles French & Co. French put in $1000 for 20 mo., Baker put in $800 for 16 mo., and Atherton put in $500 for 20 mo. They gained $1500. How many dollars of the gain ought each to receive?

5. George Jackson, William Leach, and Albert Buffington traded in company. Jackson put in $144 for 6 mo., Leach put in $72 for 7 mo., and Buffington put in $216 for 6 mo. 20 da. They gained $114. What was each man's share of the gain?

6. A, B, C, and D hired a pasture together, in which A pastured 4 cows 13 weeks, B pastured 5 cows 16 weeks, C pastured 8 cows 10½ weeks, and D pastured 4 cows 16 weeks. The rent of the pasture was $102. How many dollars ought each man to pay?

7. Samuel Austin, Jacob Brown, and Moses Sumner formed a partnership for 2 years, under the name of Samuel Austin & Co. Austin at first paid in to the stock $1000, but after 8 mo. had elapsed he paid in $500 more. Brown at first paid in $1250, and 16 mo. afterwards he paid in $250 more. Sumner at first paid in $1500, but at the end of 16 mo. he took out $500. They gained $3600. What was each man share of the gain?

Solution.

Interest of $1000 for 8 mo. = $ 40
Interest of $1500 for 16 mo. = $120
} = { $160 = int. Austin's stock.

Interest of $1250 for 16 mo. = $100
Interest of $1500 for 8 mo. = $ 60
} = { $160 = int. Brown's stock.

Interest of $1500 for 16 mo. = $120
Interest of $1000 for 8 mo. = $ 40
} = { $160 = int. Sumner's stock.

By this, it appears that the interests of their respective stocks, for the time they were in trade, were alike. Hence the gain should be divided equally, and each partner should have ½ of $3600, which is $1200.

NOTE. — Other solutions similar in character to those given to the 1st example might have been added, but as the pupil can readily discover them, they have been omitted.

8. Edward Holmes and Samuel Clapp traded in company. Holmes at first put into the firm $1000, but at the end of 9 mo. he put in $500 more, and at the end of 18 mo. he withdrew $300. Clapp at first put in $800, but at the end of 6 mo. he put in $600 more, and at the end of 15 mo. he put in $600 more. At the end of 2 years they had gained $1500. What was each partner's share of the gain?

9. Joseph Southwick, Francis Lowe, and Henry Taft formed a partnership for 3 years, under the name of Southwick, Lowe, & Taft. When they commenced business, each partner put in $3000, but at the end of the first year Southwick put in $3000 more, and Lowe withdrew $1500. At the end of the second year, Southwick withdrew $2000, and Lowe put in $4000, and Taft put in $2000. When the partnership expired, they found that they had gained $9000. What was each partner's share of the gain?

10. S. Gamwell, C. Grover, R. Wheelock, and W. Godding formed a partnership under the title of Gamwell, Grover, & Co. Gamwell at first put in $8000, but at the end of 6 mo. he withdrew $2000, and at the end of 12 mo. he withdrew $1000 more. Grover at first put in $6000, but at the end of 10 mo. he put in $3000 more. Wheelock put in $7000. Godding at first put in $10,000; at the end of 6 mo. he withdrew $2000, and at the end of 14 mo. he put in $4000. At the end of 2 years they found that they had gained $12,000. What was each man's share of the gain?

INDEX.

B. B. MUSSEY & COMPANY,

NO. 29 CORNHILL, BOSTON,

Publish, and would respectfully call attention to, the

PESTALOZZIAN SERIES OF SCHOOL ARITHMETICS,

as being the most systematic and philosophical in their arrangement, and the most inductive and progressive in their exercises, of any arithmetical works before the public. Their author, Dana P. Colburn, Principal of the Rhode Island State Normal School at Providence, is well known as a most successful and popular teacher in this department of science, having been for the past six years Teacher of Mathematics in the Massachusetts Teachers' Institutes, under the direction of the Rev. B. Sears, D. D., Secretary of the Board of Education.

PART I.—THE FIRST STEPS IN NUMBERS: by D. P. COLBURN and G. A. WALTON.

This work, designed as the pupil's first book, involves only small numbers; but, by the great variety of its exercises and simplicity of its illustrations, it gives the pupil just ideas of the nature of numbers, and skill and facility in using them; while its varied and interesting problems give him healthful intellectual exercise and valuable mental training.

PART II.—THE DECIMAL SYSTEM OF NUMBERS ILLUSTRATED AND PRACTICALLY APPLIED: by D. P. COLBURN.

This work follows the First Steps, and is a treatise on Oral and the elements of Written Arithmetic. It so presents the higher numbers as to develop the nature and exhibit the simplicity of the decimal system of representing numbers, and show the best methods of reckoning by it. It aims to give the pupil a *complete mastery* over all the elementary operations on numbers, and contains a greater number and variety of questions, problems, and exercises than any other book of the size ever published. The pupil who studies it carefully can hardly fail to acquire skill and facility in the use of figures.

PART III. will be published early in the Spring of 1855, and will be a complete Treatise on Practical Arithmetic.

B. B. MUSSEY & CO. also publish a work by the same author entitled

INTEREST, DISCOUNT, EQUATION OF PAYMENTS, &c.:

WITH NUMEROUS PROBLEMS FOR SOLUTION.

This work is designed for advanced classes in Common Schools, High Schools, and Academies, and for those who are preparing for mercantile or business pursuits. It contains the best and most approved methods of computing interest, equating accounts, and performing the other kindred problems of business life.

The following Recommendations will show in what estimation these works are held by some of our most accomplished educators: —

From N. Tillinghast, Esquire, for thirteen years Principal of Bridgewater Normal School.

BRIDGEWATER, May 11, 1852.

D. P. COLBURN, ESQ. Dear Sir: I have used your "Decimal Arithmetic" with a class of my pupils from the commencement of the Written Arithmetic to the end of the volume, and therefore feel able to speak of it with some degree of confidence.

It seems to me by far the best introduction to the science of arithmetic that I have seen. It has done more for my pupils than any book I have ever used; I have found but very little necessity for verbal explanations. I cannot well see how a scholar could go faithfully over it without acquiring a good knowledge of the principles and practice of arithmetic. N. TILLINGHAST.

Marshall Conant, Esq., Principal of the Bridgewater State Normal School, in a letter to the author, says: —

"Your books are *excellent* — *have* done, and *will* do, us excellent service."

The Assistant Teachers thus write: —

BRIDGEWATER, September 19, 1854.

The Decimal System of Numbers has been used in this school since its first appearance, and the work on Interest, &c., during two terms.

These books, by their rational manner of treating the subjects under consideration, relieve the teacher from much labor in explanation, which, with other works, is necessary to lead the pupil to any thing like an accurate knowledge of the *principles* of arithmetic. Were they generally used and *thoroughly mastered*, we should confidently expect to see the very crude notions of the science of numbers, now so prevalent, rapidly disappear. Hoping soon to welcome the remainder of the series, we remain yours truly,

EDWIN C. HEWETT, } Assistant Teachers
JAIRUS LINCOLN, JR., } State Normal School.

From George S. Houghton, Esq., Principal of the Mathematical Department of the New England Normal Institute, Lancaster, Mass.

LANCASTER, September 25, 1854.

MESSRS. B. B. MUSSEY & CO. Gentlemen: It affords me great pleasure to give my testimony to the merits of the "Decimal System of Numbers," and "Interest, Discount, and Equation of Payments." Both these works have been in use in the department under my charge during the past year, and with the most satisfactory results. The problems in them are well chosen and adapted, and the methods of solution clear, concise, and eminently practical. I consider the works as the best of the kind with which I am acquainted, and hope they may meet with the approval they so well deserve.

GEORGE S. HOUGHTON.

From the Ohio Teacher.

We have no hesitancy in pronouncing this (The Decimal System of Numbers) an excellent treatise on elementary arithmetic. It shows the riches of our decimal system, and its entire sufficiency for the supply of all requisite elementary knowledge in the science. * * * Mr. Colburn has certainly produced a work which will do much good in the hands of intelligent teachers.

From the Boston Daily Bee.

"The Decimal System of Numbers."—Such is the title of a sterling work by Dana P. Colburn. No better publication could well be conceived. It should at once be introduced into our schools. Its mastery requires just that training of the intellect which should be the boast, as it would be the credit, of every good scholar. We recognize its worth on every page, and trust that its use and circulation will be commensurate with its merits.

From D. B. Hagar, Esq., Principal of the Eliot High School, West Roxbury, and President of the Norfolk County Teachers' Association.

WEST ROXBURY, September 16, 1854.

D. P. COLBURN, ESQ. Dear Sir: Your work on Interest, Equation of Payments, &c., has been carefully tested in the Eliot School; and I have no hesitancy in saying that, in my opinion, it is decidedly superior to all similar treatises with which I am acquainted. The modes of operating laid down by you are remarkable for their brevity and clearness, and many of them are precisely those which are in use by our most rapid and accurate accountants. As especially adapted to prepare young men for efficient practice in the counting room, your work deserves the highest praise. Yours, &c.

D. B. HAGAR.

The Grammar Masters of Salem thus write:—

SALEM, September, 1854.

The undersigned, having given your book on Interest, Equation of Payments, &c., a fair trial in their schools, *do not hesitate* to pronounce it *far superior* to any thing of the kind with which they are acquainted, both as regards the *character of its examples* and the *methods of their solution.* They think the BOOK ITSELF its *best* recommendation, and that it only need *be known to command favor.*

J. F. BROWN, Principal of the Browne School.
WM. P. HAYWARD, Principal of the Epes School.
N. A. MOULTON, Principal of the Hacker School.
A. J. MANCHESTER, Principal of the Phillips School.

From Richard Edwards, Esq., Principal of the State Normal School, Salem, Mass.

SALEM, September 28, 1854.

D. P. COLBURN, ESQ. Dear Sir: I have had an opportunity of testing the value of your work on Interest and Discount as a school book, having used it in the English High School of this city, and I do not hesitate to pronounce it superior to any other book on these subjects that is known to me. It presents processes and principles instead of the mere words of an arbitrary rule. The pupil is made acquainted with realities, and not merely with the signs of things. The brevity and practical character of the processes are also very striking. There is nothing in it which the pupil must unlearn when he goes into the counting room. Besides this, it has much valuable information not contained in ordinary treatises. I am expecting it to do much to improve the methods of teaching in our schools. We have introduced it into the Normal School.

Very truly yours, RICHARD EDWARDS.

From Teachers in Fall River.

FALL RIVER, September 20, 1854.

MR. COLBURN. Dear Sir: I am using your "Interest, Discount, and Equation of Payments," and I take pleasure in expressing the satisfaction it has given me. I find the problems very practical, choice, and well adapted to the

wants of the pupil. I consider the book eminently worthy of patronage, and hope that it may meet with the favor its merit deserves.

Yours very truly, GEORGE G. LYON,
Principal of Anawan Street Grammar School.

We have used your work on "Interest, Discount," &c., in our schools for several months, and fully concur in the opinion expressed by Mr. Lyon in regard to its merits.

ALBION K. SLADE,
Principal of June Street Grammar School.
WM. R. GORDAN,
Principal of ——— Grammar School.

From the Massachusetts Teacher.

INTEREST, DISCOUNT, EQUATION OF PAYMENTS, &c.; WITH NUMEROUS PROBLEMS FOR SOLUTION. By Dana P. Colburn.

This is the title of an excellent little Hand Book for teachers and for advanced classes in arithmetic. The beauty of the work consists in its adaptation to the wants and circumstances of business men. It contains, what no work on arithmetic has professed to give, the ready and expeditious methods of computing interest which the best accountants are constantly using. Equation of payments and of accounts is presented in such a manner that the youth who studies the subject from this work may fit himself for entering the counting room with entire confidence in his ability to meet all questions which may come before him.

Most of our modern arithmetics present a great improvement in this department over the old ones; but none of them explains the somewhat intricate subject of equating accounts so satisfactorily, nor offers so good a supply of examples, as the work of Mr. Colburn. The remarks in section 10th on computing time are worthy the especial attention of teachers. We can recommend it in the full confidence that it will give entire satisfaction, knowing, too, that it supplies a great deficiency.

The following votes, passed by the School Committee of Lawrence and signed by them, have been received:—

Voted, That, from a careful examination of "The First Steps in Numbers" by Messrs. Colburn and Walton, we consider the work admirably adapted to the end for which it was prepared. It is so strictly and thoroughly elementary, its arrangement is so systematic, its steps are so gradual, and the pupil is so well drilled upon the various combinations of simple numbers, that we think he must understand what he passes over, be consequently interested in its study, and well prepared to ascend the higher steps in numbers. Therefore,—

Voted, That said work be adopted as a Text Book in the public schools of this town.

(Signed) GEORGE PACKARD,
L. WHITING,
N. W. HARMON,
HENRY F. HARRINGTON,
JAMES D. HERRICK,
} School Committee.

LAWRENCE, October 3, 1854.

Dear Sir: You ask me for a candid opinion of your First Steps in Numbers. I will give it.

It has been in use in this city for several years; and I am perfectly satisfied that not only is it *comparatively* superior as a manual, but it *excellently well* supplies what is demanded of such a work. Through its instrumentality our

children in the lower grades of schools are well grounded in arithmetical knowledge, and its lucid method and thorough treatment carry the mind much farther than its unpretending dimensions might lead one to anticipate.

I believe that it is highly valued by the School Committee.

Truly yours, HENRY F. HARRINGTON,
Superintendent of Schools.

BRIDGEWATER, October 4, 1854.

I have examined with interest and care the work by D. P. Colburn, Esq., entitled "Interest, Discount, Equation of Payments, &c." It seems to me extremely well drawn up, and well fitted to suggest to scholars and accountants some of the best methods of calculation in the subjects which it comprises.

N. TILLINGHAST.

From Thomas Sherwin, Esq., author of Sherwin's Algebra, and Principal of English High School, Boston.

I have perused with much pleasure your "First Steps in Numbers," and am happy to give it my hearty commendation. The work contains several important principles not commonly introduced into primary arithmetic, and the exercises are well adapted to give the learner a thorough knowledge of these principles and great facility in the application of them. I hope your treatise will be introduced into general use.

Very respectfully yours, THOMAS SHERWIN.

From Professor Samuel S. Greene, now Superintendent of Public Schools, Providence, R. I.

MESSRS. COLBURN AND WALTON: I have examined your "First Steps in Numbers." It is an excellent work, and is well adapted to beginners. Its merits consist in the thoroughness of the drill to which it subjects the pupil, in the clear and gradual development of the elementary processes of numerical calculation, and in admirably preparing the way for written arithmetic.

SAMUEL S. GREENE,
Principal of the Phillips Grammar School, Boston.

In a note of August 31, 1840, *George B. Emerson, Esq., says:—*

I have looked over your work with interest, and cannot but think it very valuable. It takes up numbers in a way different from that of the author of First Lessons, and supplies some deficiencies, especially as introductory to the use of the slate. * * * I believe that the faithful use of the "First Steps" will be an excellent introduction to the knowledge of arithmetic.

A lady, writing from Wells River, Vermont, under date of July 1, 1852, *says of the First Steps:—*

I cannot tell you how delighted I am with that book. It has thrown a new charm around the subject of arithmetic. Two scholars that I gave up last winter as almost hopeless are no longer dull in figures, but look forward as eagerly to the hour of recitation as to the hour of play.

From the Scholar's Gazette.

It is a difficult thing to make an arithmetic out of which, and according to which, children can be taught the nature and use of numbers successfully. The one before us, the work of two experienced teachers who are evidently masters

of their subject, is laid before the public, aiming to be such a treatise; and, from a careful examination of its pages, it appears to possess merits equal to the design of the authors. Being the *first steps* in numbers, it takes the pupil at 1 and leads him on with a skilful hand, step by step, up the arithmetical stairs in all conceivable ways, keeping him so long and drilling him so thoroughly upon each that he cannot fail to get a good knowledge of the ground he travels. The book is perfectly systematic throughout. Its arrangement is most careful and correct. Its examples are interestingly and happily written. It seems to be a *complete* elementary arithmetic, admirably calculated to give the pupil a clear understanding of the operations to be performed with numbers to the extent which it reaches. We take pleasure in calling the attention of teachers to this work, which has already been introduced into several important towns in the State.

THE PESTALOZZIAN SERIES OF

SCHOOL ARITHMETICS.

Part I.

THE FIRST STEPS IN NUMBERS.

By D. P. COLBURN and G. A. WALTON.

Part II.

THE DECIMAL SYSTEM OF NUMBERS,

ILLUSTRATED AND PRACTICALLY APPLIED.

By DANA P. COLBURN.

Part III.

Is in preparation, and will be published early in the Spring of 1855.

RECOMMENDATIONS.

From J. W Upton, Esq., now Teacher of English Department of Pinkerton Academy, Derry, N. H.

GREENFIELD, February 26, 1850.

Gentlemen: During the present term of my school, my assistant has used the copy of your Arithmetic which you had the kindness to present me. I find that the scholars (though not provided with books) have attained a greater facility in the performance of examples in the different text books used by them in their several classes; that no other arithmetic takes so well with them; that more interest is manifested in recitation; and that the mental discipline is not only good, but practical.

Satisfied that much has been done with one book, and that if our scholars can each have a book of the same kind more will be done, I shall earnestly endeavor to have our committee adopt your Arithmetic as a text book.

Yours respectfully, JOSEPH W. UPTON.

From N. Tillinghast, Esq., Principal of Bridgewater Normal School.

BRIDGEWATER, May 11, 1852.

D. P. COLBURN, ESQ. Dear Sir: I have used your "Decimal Arithmetic" with a class of my pupils from the commencement of the Written Arithmetic to the end of the volume, and therefore feel able to speak of it with some degree of confidence.

It seems to me by far the best introduction to the science of arithmetic that I have seen. It has done more for my pupils than any book I have ever used. I have found but very little necessity for verbal explanations. I cannot well see how a scholar could go faithfully over it, without acquiring a good knowledge of the principles and practice of arithmetic. N. TILLINGHAST.

From the Boston Daily Bee.

"THE DECIMAL SYSTEM OF NUMBERS." Such is the title of a sterling work by Dana P. Colburn. No better publication could well be conceived. It should at once be introduced into our schools. Its mastery requires just that training of the intellect which should be the boast, as it would be the credit of every good scholar. We recognize its merit and worth in every page, and trust that its use and circulation will be commensurate with its merits.